New Supersymmetric and Stable, Non-Supersymmetric Phases in Supergravity and Holographic Field Theory

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Abstract

We establish analytically that the potential of $\mathcal{N}=8$ supergravity in four dimensions has a new $\mathcal{N}=1$ supersymmetric critical point with $U(1)\times U(1)$ symmetry. We work within a consistent $\mathcal{N}=1$ supersymmetric truncation and obtain the holographic flow to this new point from the $\mathcal{N}=8$ critical point. The operators that drive the flow in the dual field theory are identified and it is suggested the new critical point might represent a new conformal phase in the holographic fermion droplet models with sixteen supersymmetries. The flow also has $\frac{c_{\rm IR}}{c_{\rm UV}}=\frac{1}{2}$. We examine the stability of all twenty known critical points and show that the $SO(3)\times SO(3)$ point is a perturbatively stable non-supersymmetric fixed point. We also locate and describe a novel critical point that also has $SO(3)\times SO(3)$ symmetry and is related to the previously known one by triality in a similar manner to the way that the $SO(7)^{\pm}$ points are related to one another.

Contents

1 Introduction

In the early 1980's it was hoped that gauged $\mathcal{N} = 8$ supergravity would provide a theory of unified gauge interactions that was embedded within a sensible theory of quantum gravity. This hope went unfulfilled for a number of fundamental reasons relating to the problems of getting a viable vacuum with the appropriate chiral fermions. One of the issues was that the natural vacuum states all seemed to have Planck-scale, negative cosmological constants. Thus $\mathcal{N} = 8$ supergravity seemed to be an interesting, extremely symmetric but rather rigid and unphysical theory.

The advent of holography changed this state of affairs precisely because gauged $\mathcal{N}=8$ supergravity is the field theory of the massless modes in the gravitational background generated by a stack of $M2$ branes. The AdS vacua and related solutions could then be reinterpreted in terms of conformal fixed points, perturbed fixed points and flows in the holographically dual field theory on the $M2$ branes. For the maximally symmetric ($\mathcal{N}=8$) vacuum, there is a precise holographic dictionary that states that the fields of the $\mathcal{N} = 8$ supergravity are dual to relevant operators in the energy-momentum tensor supermultiplet of the dual gauge theory. In particular, $\mathcal{N} = 8$ supergravity contains fields that are dual to fermion and boson bilinears in the M-brane field theory and so the supergravity theory can be used to study holographic flows that are driven by mass terms and vevs of such bilinears. More recently, the deeper understanding of the field theory on the $M2$ brane [1, 2, 3] enabled the study of the gravity-gauge duality from both sides of the duality. The interest in gauged $\mathcal{N} = 8$ supergravity has also grown considerably due to the extensive activity in AdS/CMT where the corresponding holographic field theories might be used to study interesting, strongly coupled condensed matter systems in $(2 + 1)$ dimensions.

There have been two rather different approaches to the study of AdS/CMT: The "bottom-up" (or "phenomenological") approach and the "top-down" approach. In the former, the gravity dual of an interesting condensed matter system is postulated *ab initio* (see, for example, [4, 5, 6]), without using the more well-established holographic field theories and their gravity duals. In the latter approach one tries to realize interesting phenomenological models within theories that have well-established holographic dictionaries, as one does in M-theory or IIB supergravity (see, for example, [7]–[12]). It is very important to do this, not only because of the dictionary, but also to make sure that the complete holographic theory does not have other low-mass modes that compromise or destroy the effect that one finds in the reduced or effective field theory. Indeed, there is at least one example of a consistent, even supersymmetric theory with a (non-supersymmetric) ground state that is completely stable within that consistent truncation but is pathologically unstable within the complete holographic theory [13].

Stability of AdS vacua in supergravity has been well understood for many years. The supergravity potentials are generally unbounded below and do not even have local minima but merely

have critical points at negative values of the potential. The negative values of the potential lead to anti-de Sitter (AdS) vacua that are expected to be dual to non-trivial conformal fixed points in the field theory. In such a vacuum one can tolerate a certain amount of negative mass because the gravitational back-reaction can stabilize the fluctuations [14]. To be more precise, suppose that there is a scalar field, ϕ, in d-dimensions with a potential, $\mathcal{P}(\phi)$, that has a critical point at ϕ_0. Taking the Lagrangian to be

$$e^{-1}\mathcal{L} \;=\; \tfrac{1}{2}R - \tfrac{1}{2}(\partial_\mu\phi)^2 \;-\; \mathcal{P}(\phi)\,, \tag{1.1}$$

then the AdS_d vacuum at ϕ_0 is stable to quadratic fluctuations if [14, 15]

$$\frac{(d-1)(d-2)}{2}\left(\frac{\mathcal{P}''(\phi_0)}{\mathcal{P}(\phi_0)}\right) \;\le\; \frac{(d-1)^2}{4}\,. \tag{1.2}$$

For more general Lagrangians with more scalars and more complicated kinetic terms, one expands quadratically, normalizes to the form of (1.1) and then applies the Breitenlohner-Freedman (BF) bound, (1.2), to the eigenvalues of the quadratic fluctuation matrix for the potential. In the holographically dual field theory, violating the bound, (1.2), shows up as a manifest pathology: The holographic dictionary shows that such perturbations are dual to operators with complex conformal dimensions.

Supersymmetry guarantees stability through the usual energy bound arguments applied to anti-de Sitter space [16, 17]. This implies complete classical and semi-classical stability, and not merely to solutions based upon critical points. Supersymmetric flow solutions are therefore completely stable but imposing supersymmetry also significantly restricts the physics. Indeed, in its simplest form, superfluidity and superconductivity require the formation of a fermion condensate and at first sight it seems unclear whether such a condensation alone could be rendered supersymmetric. It is also not immediately obvious how a Fermi-sea would ever develop in a supersymmetric ground state. On the other hand, we know from [18] that there are holographic flows that have *sixteen* supersymmetries and whose infra-red fixed point is described by free fermions and excitations of the Fermi sea. We will show that our new supersymmetric solutions and flows are closely related to these holographic fermion theories and that the new critical point might have the interpretation of a state in this theory.

Rather remarkably, the non-supersymmetric $SO(3)\times SO(3)$-invariant critical point discussed in [19] has not been subjected to a full stability analysis. This critical point is also part of the $\mathcal{N}=5$ truncation of $\mathcal{N}=8$ supergravity, where it has an $SO(3)$ invariance. In this setting it was discussed in [20, 17] and was shown to be perturbatively stable within the $\mathcal{N}=5$ theory [14, 21]. Moreover a positive mass theorem was established within the $\mathcal{N}=5$ theory [22] for this critical point and so it was shown to be completely semi-classically stable. This is not sufficient to guarantee perturbative stability within the full $\mathcal{N}=8$ theory because other fields

could destabilize the vacuum, as in [13], but we will show here that all the scalar fluctuations around this point satisfy the BF bound and so this point is the first known, perturbatively stable, non-supersymmetric AdS vacuum for the $\mathcal{N}=8$ theory.[1] It would be interesting to see if the arguments of [22] could be extended to the complete $\mathcal{N}=8$ theory to establish complete semi-classical stability.

As a result of all these factors it was, and continues to be, very important to classify the solutions of gauged $\mathcal{N}=8$ supergravity in four dimensions and, most particularly, find the critical points of the scalar potential of that theory and look at their stability. A systematic analytic method for computing the potential was developed a long time ago [19] and was exploited in [21] to obtain a variety of interesting new supersymmetric and non-supersymmetric vacua. The limitation of this approach is that analytic computations generally require a fairly high level of symmetry in order for the explicit computations to be possible. Thus, for over 25 years, the only known critical points of the $\mathcal{N}=8$ potential had at least $SU(3)$ or $SO(3)\times SO(3)$ symmetry. On the other hand, a numerical approach to the problem that utilizes sensitivity back-propagation to obtain fast high quality gradients recently has revealed that this potential has many more critical points [24, 25] with relatively low levels of symmetry. The fourteen new points discovered in [24] have at most $U(1)\times U(1)$ symmetry and eight of them have no residual continuous symmetry at all. Given this low level of symmetry, it is highly unlikely, and probably impossible for analytic searches to have discovered them. (Indeed, in Appendix B.3 we give a first glimpse of the difficulty of this problem.) Moreover, even if one knows exactly where these critical points lie, the scalar expectation values are generically so complicated that the $E_{7(7)}$ matrix has a characteristic polynomial of very high degree and this renders analytic exponentiation computationally out of reach.

One of the important discoveries in [24] was the possibility of a new $\mathcal{N}=1$ supersymmetric critical point with $U(1)\times U(1)$ symmetry and with $\mathcal{P}=-12g^2$. This point is potentially very interesting for several reasons. First, it is a new supersymmetric point. Second, the $U(1)\times U(1)$ symmetry commutes with an $SO(4)\subset SO(8)$ gauge symmetry and this means that there are a rich collection of possible gauge fields and hence chemical potentials that could induce a flow to this critical point. Moreover, the value of the cosmological constant (dictated by the value of $\mathcal{P}$) is less than that of all the critical points that have, at least, an $SU(3)$ invariance [21]. The c-theorem [26] implies that the cosmological constant must decrease along holographic flows thus interesting, but unstable, flows in AdS/CMT, like some of those considered in [10, 12], might be arranged to ultimately flow to this new $\mathcal{N}=1$ supersymmetric critical point. Finally, as we will

[1]This finally and definitively invalidates the "folk theorem" that suggest that only supersymmetric critical points are stable in maximal supergravity theories in more than three dimensions. BF stability without supersymmetry in a maximally supersymmetric theory was first observed in the three-dimensional, $SO(8)\times SO(8)$ gauged $\mathcal{N}=16$ Chern-Simons model: See formula (5.8) in [23].

discuss, this new supergravity phase may well have an interesting holographic interpretation in terms of a new phase of the fermion droplet model considered in [18].

Fortunately, since one now knows where to look, this point can be constructed analytically within a consistent truncation of the $\mathcal{N}=8$ theory and in this paper we construct this truncation, give an analytic description of the point showing that it does indeed have precisely the symmetries suggested in [24]. We also give a superpotential on the truncated sector and obtain the supergravity flow to the new $\mathcal{N}=1$ point from the $\mathcal{N}=8$ point.

In Section 2 we discuss successive consistent truncations of the $\mathcal{N}=8$ theory down to a simple $\mathcal{N}=1$ theory that we will then analyze in detail. We include some intermediate $\mathcal{N}=4$ and $\mathcal{N}=2$ intermediate truncations that not only make the ultimate truncation clearer but should also prove valuable if one wishes to include vector multiplets so as to include holographic chemical potentials. In Section 3 we construct the scalar potential and superpotential of this $\mathcal{N}=1$ theory and show that the potential has critical points corresponding to the non-supersymmetric $SO(3)\times SO(3)$-invariant point and to the new $\mathcal{N}=1$ supersymmetric point found in [24]. The superpotential only has the maximally supersymmetric critical point and the $\mathcal{N}=1$ supersymmetric critical point. We also obtain the holographic flow equations for this superpotential and exhibit the $\mathcal{N}=1$ supersymmetric flow from the $\mathcal{N}=8$ point to the $\mathcal{N}=1$ point.

In Section 4 we consider the UV asymptotics of the holographic flow and identify the non-normalizable and normalizable fields involved. It turns out that the only non-normalizable field involved is the one that was extensively studied in [27, 28, 18] and whose holographic field theory may be described, at least in the infra-red, in terms of free fermions in $(1+1)$-dimensions. The flow we consider here is rather more complicated and involves modes that break a lot of symmetry and that were not considered in [27, 28, 18], however, these extra modes are *normalizable* and this strongly suggests that the flow here might represent some exotic supersymmetric state of the fermion droplet model defined in [18].

In Section 5 we examine the $SO(3)\times SO(3)$ invariant point in more detail, show that it is stable and discuss possible flows to this point. In Section 6, we discuss the analytic properties of a *novel* and different $SO(3)\times SO(3)$ symmetric critical point that is unstable. In Section 7 we consider all the other non-supersymmetric critical points and discuss their mass spectrum. Out of the eighteen known, non-supersymmetric critical points, it is only the $SO(3)\times SO(3)$-invariant point with $\mathcal{P}=-14g^2$ that satisfies the BF bound and is thus perturbatively stable. We also discuss the prospects of finding more critical points and describe why we believe that there are probably many more critical points with very low levels of symmetry. We also describe why such points are almost certainly outside the reach of analytic algorithms but are likely to be accessible using numerical searches. Finally, in Section 8, we make some remarks about the broader implications of our work.

2 Consistent truncations

To examine the new supersymmetric point, we will describe it as part of an $\mathcal{N}=1$ supersymmetric truncation of the $\mathcal{N}=8$ theory. We will, however, arrive at this $\mathcal{N}=1$ truncation successively by reducing to sectors that are invariant under progressively more symmetry. Indeed, we will first pass to an $\mathcal{N}=4$ theory and then show that the $\mathcal{N}=1$ theory lies in the common overlap of two different $\mathcal{N}=2$ theories. We anticipate that these intermediate $\mathcal{N}=2$ truncations will be useful when one wishes to study more general supergravity flows that involve vector fields and induce chemical potentials in the holographic field theory.

2.1 An intermediate $\mathcal{N}=4$ theory

Since the critical point we seek is invariant under under the two rotations, $\mathcal{R}_{36}$ and $\mathcal{R}_{45}$, that generate an $SO(2)^2$ subgroup of $SO(8)$ [24, 25], the obvious first step is to consider the sector of the entire $\mathcal{N}=8$ theory that is invariant under this symmetry.

These rotations preserve four supersymmetries, $\varepsilon^1, \varepsilon^2, \varepsilon^7$ and ε^8, and so the result must be $\mathcal{N}=4$ supersymmetric. Indeed it is $\mathcal{N}=4$ supergravity coupled to two $\mathcal{N}=4$ vector multiplets: the six vector fields in the graviton multiplet are A_μ^{IJ}, $I, J \in \{1,2,7,8\}$ and the two vector multiplets are generated by A_μ^{36} and A_μ^{45}.

To isolate the scalar manifold, consider the $SO(4)_A \times SO(4)_B$ subgroup of $SO(8)$ where $SO(4)_A$ acts on indices (1278) and $SO(4)_B$ acts on indices (3456). Inside $E_{7(7)}$, $SO(4)_B$ commutes with $SU(4) \times SL(2,\mathbb{R})$ where the $SU(4) \subset SU(8)$ acts on the (1278) and the non-compact generators of the $SL(2,\mathbb{R})$ can be represented by the self-dual form:

$$\Upsilon_0 \equiv (w_0\, dx_1 \wedge dx_2 \wedge dx_7 \wedge dx_8 + \overline{w}_0\, dx_3 \wedge dx_4 \wedge dx_5 \wedge dx_6)\,, \tag{2.3}$$

where w_0 is a complex number. This $SL(2,\mathbb{R})$ defines the complex scalar of the $\mathcal{N}=4$ graviton supermultiplet. The commutant of the $SO(2)^2$ generated by $\mathcal{R}_{36}$ and $\mathcal{R}_{45}$ in $E_{7(7)}$ is simply $SO(6,2) \times SL(2,\mathbb{R})$ where $SU(4) \cong SO(6)$ is embedded in the obvious manner. The non-compact generators of the $SO(6,2)$ can be represented by the self-dual forms:

$$\begin{aligned}
\Upsilon_1 &\equiv (z_1\, dx_1 \wedge dx_3 \wedge dx_6 \wedge dx_7 - \bar{z}_1\, dx_2 \wedge dx_4 \wedge dx_5 \wedge dx_8)\,,\\
\Upsilon_2 &\equiv (z_2\, dx_1 \wedge dx_4 \wedge dx_5 \wedge dx_7 - \bar{z}_2\, dx_2 \wedge dx_3 \wedge dx_6 \wedge dx_8)\,,\\
\Upsilon_3 &\equiv (z_3\, dx_1 \wedge dx_4 \wedge dx_5 \wedge dx_8 + \bar{z}_3\, dx_2 \wedge dx_3 \wedge dx_6 \wedge dx_7)\,,\\
\Upsilon_4 &\equiv (z_4\, dx_1 \wedge dx_3 \wedge dx_6 \wedge dx_8 + \bar{z}_4\, dx_2 \wedge dx_4 \wedge dx_5 \wedge dx_7)\,,\\
\Upsilon_5 &\equiv (z_5\, dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_6 + \bar{z}_5\, dx_4 \wedge dx_5 \wedge dx_7 \wedge dx_8)\,,\\
\Upsilon_6 &\equiv (z_6\, dx_1 \wedge dx_2 \wedge dx_4 \wedge dx_5 + \bar{z}_6\, dx_3 \wedge dx_6 \wedge dx_7 \wedge dx_8)\,.
\end{aligned} \tag{2.4}$$

This defines the twelve scalars in the two $\mathcal{N}=4$ vector multiplets.

2.2 Two intermediate $\mathcal{N}=2$ theories

One can now use some discrete subgroups of $SO(8)$:

$$g_1: \qquad (x_1, x_2, x_3, x_4) \leftrightarrow -(x_1, x_2, x_3, x_4), \tag{2.5}$$

$$g_2: \qquad (x_1, x_3, x_6, x_7) \leftrightarrow (-x_1, x_4, x_5, x_8), \tag{2.6}$$

with the remaining x_j invariant. Note that these transformations have determinant one and so live in $SO(8)$. More importantly, these discrete symmetries leave invariant the scalars that define the new $\mathcal{N}=1$ critical point.

The two intermediate $\mathcal{N}=2$ theories are defined by requiring invariance under either g_1 or g_2 separately.

Invariance under g_1 leaves the two supersymmetries, ε^7 and ε^8 and only two vector fields: A_μ^{78} and A_μ^{12}. From the supersymmetries it is evident that A_μ^{78} must lie in the $\mathcal{N}=2$ graviton multiplet while A_μ^{12} must generate an $\mathcal{N}=2$ vector multiplet. The scalars in the $SL(2,\mathbb{R})$ defined by (2.3) belong to this vector multiplet. The remaining fields then make up two $\mathcal{N}=2$ hypermultiplets and these contain the four complex g_1-invariant scalars in (2.4) parametrized by z_1, z_2, z_3 and z_4 in (2.4). Thus the complete scalar manifold can be described in terms of the coset:

$$\frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SU(2,2)}{SU(2)\times SU(2)\times U(1)}\,. \tag{2.7}$$

Invariance under g_2 leaves the two supersymmetries, ε^2 and $\varepsilon^7+\varepsilon^8$ and three vector fields: $B_\mu^{(0)} \equiv A_\mu^{27} + A_\mu^{28}$, $B_\mu^{(1)} \equiv A_\mu^{17} - A_\mu^{18}$ and $B_\mu^{(2)} \equiv A_\mu^{36} + A_\mu^{45}$. From the supersymmetries it is evident that $B_\mu^{(0)}$ must lie in the graviton multiplet while $B_\mu^{(1)}$ and $B_\mu^{(2)}$ generate two $\mathcal{N}=2$ vector multiplets. Again, the scalars in the $SL(2,\mathbb{R})$ defined by (2.3) belong to one of these vector multiplets. There are three complex g_2-invariant scalars that arise from (2.4) by taking $z_3 = -z_1$, $z_4 = -z_2$, $z_6 = -z_5$. One of these belongs to a vector multiplet while the remainder constitute an $\mathcal{N}=2$ hypermultiplet. Thus the complete scalar manifold can be described in terms of the coset:

$$\frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SU(2,1)}{SU(2)\times U(1)}\,. \tag{2.8}$$

2.3 Defining and parametrizing the $\mathcal{N}=1$ theory

Requiring invariance under both g_1 and g_2 leaves only one supersymmetry, $\varepsilon^7+\varepsilon^8$, and projects out all the vector fields. The $SL(2,\mathbb{R})$ of the $\mathcal{N}=4$ graviton multiplet survives while, in (2.4), one must set $z_5 = z_6 = 0$ and $z_3 = -z_1$ and $z_4 = -z_2$. This reduces $SO(6,2)$ to $SO(2,2) \cong SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ and so the scalar coset is now:

$$\frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SL(2,\mathbb{R})}{SO(2)}\,. \tag{2.9}$$

These simply form three $\mathcal{N}=1$ scalar multiplets with the g_1-invariant and g_2-invariant fermions:

$$\chi^{127} - \chi^{128}\,, \qquad \chi^{136} - \chi^{145}\,, \qquad \chi^{236} + \chi^{245}\,. \tag{2.10}$$

We are thus dealing with $\mathcal{N}=1$ supergravity coupled to three scalar multiplets.

We will work with this theory and we will use the following explicit parametrization of the $SL(2,\mathbb{R})^3$:

$$\Sigma_0 \equiv \left(w_0\, dx_1 \wedge dx_2 \wedge dx_7 \wedge dx_8 + \bar{w}_0\, dx_3 \wedge dx_4 \wedge dx_5 \wedge dx_6\right), \tag{2.11}$$

$$\begin{aligned}\Sigma_1 \equiv\ & (w_1 + w_2)\left(dx_1 \wedge dx_3 \wedge dx_6 \wedge dx_7 - dx_1 \wedge dx_4 \wedge dx_5 \wedge dx_8\right) \\ & - (\overline{w}_1 + \overline{w}_2)\left(dx_2 \wedge dx_4 \wedge dx_5 \wedge dx_8 + dx_2 \wedge dx_3 \wedge dx_6 \wedge dx_7\right),\end{aligned} \tag{2.12}$$

$$\begin{aligned}\Sigma_2 \equiv\ & (w_1 - w_2)\left(dx_1 \wedge dx_4 \wedge dx_5 \wedge dx_7 - dx_1 \wedge dx_3 \wedge dx_6 \wedge dx_8\right) \\ & - (\overline{w}_1 - \overline{w}_2)\left(dx_2 \wedge dx_3 \wedge dx_6 \wedge dx_8 + dx_2 \wedge dx_4 \wedge dx_5 \wedge dx_7\right),\end{aligned} \tag{2.13}$$

where we have set $z_1 = (w_1 + w_2)$ and $z_2 = (w_1 - w_2)$. The three commuting $SL(2,\mathbb{R})$'s are then parametrized by each of the w's. We will use the polar parametrization with:

$$w_0 = \frac{1}{2\sqrt{2}}\,\lambda_0\, e^{i\varphi_0}\,, \qquad w_1 = \frac{1}{4\sqrt{2}}\,\lambda_1\, e^{i\varphi_1}\,, \qquad w_2 = \frac{1}{4\sqrt{2}}\,\lambda_2\, e^{i\varphi_2}\,, \tag{2.14}$$

and then use the exponentiated coordinates:

$$\zeta_0 = \tanh\left(\tfrac{1}{\sqrt{2}}\,\lambda_0\right) e^{-i\varphi_0}\,, \qquad \zeta_1 = \tanh\left(\tfrac{1}{2}\,\lambda_1\right)\, e^{-i\varphi_1}\,, \qquad \zeta_2 = \tanh\left(\tfrac{1}{2}\,\lambda_2\right) e^{+i\varphi_2}\,. \tag{2.15}$$

2.4 The scalar action

The kinetic terms of the $\mathcal{N}=8$ theory reduce to precisely what one would expect of an $(SL(2,\mathbb{R}))^3$ coset:

$$\begin{aligned} e^{-1}\mathcal{L}_{kin.} &= -\frac{1}{96}\mathcal{A}_\mu{}^{ijkl}\mathcal{A}^\mu{}_{ijkl} \\ &= \frac{1}{2}\sum_{j=0}^{2}\big(\partial_\mu\lambda_j\big)^2 + \frac{1}{4}\sinh^2(\sqrt{2}\lambda_0)(\partial_\mu\varphi_0)^2 + \frac{1}{2}\sum_{j=1}^{2}\sinh^2(\lambda_j)(\partial_\mu\varphi_j)^2 \\ &= \frac{\big|\partial_\mu\zeta_0\big|^2}{(1-|\zeta_0|^2)^2} + 2\sum_{j=1}^{2}\frac{\big|\partial_\mu\zeta_j\big|^2}{(1-|\zeta_j|^2)^2}\,, \end{aligned} \tag{2.16}$$

where the normalizations are determined by the embedding indices of the $SL(2,\mathbb{R})$'s inside $E_{7(7)}$.

The scalar potential in the $\mathcal{N}=8$ theory is given by:

$$\begin{aligned} \mathcal{P} &= -g^2\big(\tfrac{3}{4}\left|A_1{}^{ij}\right|^2 - \tfrac{1}{24}\left|A_{2i}{}^{jkl}\right|^2\big) \\ &= -g^2\Big(4\cosh\lambda_1\cosh\lambda_2 + 2\cosh(\sqrt{2}\lambda_0) \\ &\qquad -\tfrac{1}{2}\sinh^2\lambda_1\sinh^2\lambda_2\left[\cosh(\sqrt{2}\lambda_0)\,(1-\cos(2\varphi_1)\cos(2\varphi_2))\right. \\ &\qquad\qquad \left. -\sinh(\sqrt{2}\lambda_0)(\cos(2\varphi_1)-\cos(2\varphi_2))\cos\varphi_0\right]\Big)\,, \end{aligned} \tag{2.17}$$

This potential has exactly three inequivalent critical points. The maximally supersymmetric vacuum:

$$\lambda_0 = \lambda_1 = \lambda_2 = 0\,; \qquad \mathcal{P} = -6\,g^2\,. \tag{2.18}$$

The other two points are:

$$\lambda_0 = 0\,, \quad \lambda_1 = \lambda_2 = \pm\log(2+\sqrt{5})\,; \quad \varphi_1 = \varphi_2 = \frac{\pi}{4}\,, \qquad \mathcal{P} = -14\,g^2\,, \tag{2.19}$$

and

$$\begin{aligned} \tfrac{1}{\sqrt{2}}\lambda_0 &= \log\Big[\frac{1}{\sqrt{2}}(\sqrt{3}\pm 1)\Big] \approx \pm 0.65848\,, \quad \lambda_1 = \lambda_2 = \log\big[\sqrt{3}\pm\sqrt{2}\big] \approx \pm 1.146216\,, \\ \varphi_0 &= \pm\frac{\pi}{2}\,, \quad \varphi_1 = \varphi_2 = \frac{\pi}{4}\,; \qquad \mathcal{P} = -12\,g^2\,. \end{aligned} \tag{2.20}$$

In terms of the complex coordinates, these are:

$$\zeta_0 = 0\,, \qquad \zeta_1 = \pm\frac{1}{2}(\sqrt{5}-1)(1-i)\,, \qquad \zeta_2 = \pm\frac{1}{2}(\sqrt{5}-1)(1+i)\,, \qquad \mathcal{P} = -14\,g^2\,, \tag{2.21}$$

and

$$\zeta_0 = \mp\frac{1}{\sqrt{3}}\,i\,, \quad \zeta_1 = \pm\frac{1}{2}(\sqrt{3}-1)(1-i)\,, \quad \zeta_2 = \pm\frac{1}{2}(\sqrt{3}-1)(1+i)\,, \quad \mathcal{P} = -12\,g^2\,. \tag{2.22}$$

The former is the non-supersymmetric $SO(3)\times SO(3)$ invariant critical point discovered in [19], with the $SO(3)$'s acting on the coordinates (x_3, x_6, x_7) and (x_4, x_5, x_8). The second point is one of the new critical points that was recently discovered numerically in [24, 25]. As we will see, this point is indeed supersymmetric, as was strongly suggested by the numerical computations. The contours of the scalar potential for $\lambda_1 = \lambda_2$, $\varphi_0 = -\frac{\pi}{2}$ and $\varphi_1 = \varphi_2 = \frac{\pi}{4}$ are shown in the first graph in Fig. 1.

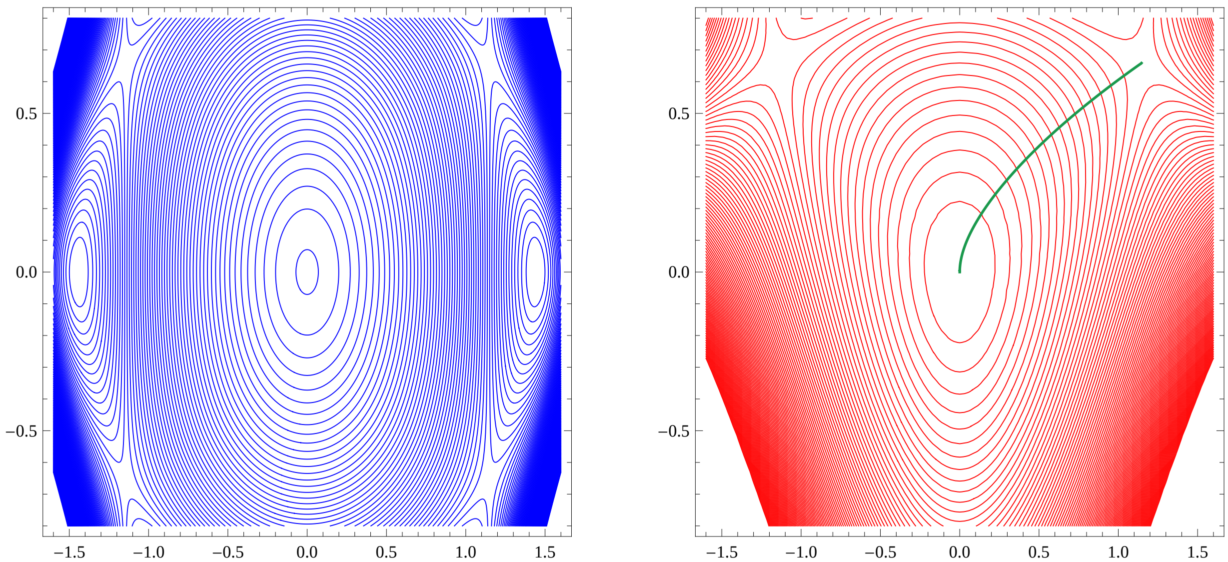

Figure 1: Contours of the scalar potential and superpotential for $\lambda_1 = \lambda_2$, $\varphi_0 = -\frac{\pi}{2}$ and $\varphi_1 = \varphi_2 = \frac{\pi}{4}$. The vertical axis is λ_0 and the horizontal axis is $\lambda_1 = \lambda_2$. The central critical point is maximally supersymmetric. The two critical points with $\lambda_0 = 0$ and $\lambda_1 \neq 0$ are copies of the $SO(3) \times SO(3)$ non-supersymmetric critical point. The four other critical points of the potential preserve $\mathcal{N}=1$ supersymmetry, but the pair with $\lambda_0 < 0$ preserves $\varepsilon^7 - \varepsilon^8$ while the pair with $\lambda_0 > 0$ preserves $\varepsilon^7 + \varepsilon^8$. The second contour plot shows the superpotential that arises from preserving $\varepsilon^7 + \varepsilon^8$ and thus only has two non-trivial (and equivalent) critical points. The curving line on the plot of the superpotential shows the $\mathcal{N}=1$ supersymmetric flow from the maximally supersymmetric point to the $\mathcal{N}=1$ supersymmetric critical point.

2.5 The superpotential

Define the complex superpotential by:

$$\begin{aligned}
\mathcal{W} \;&=\; \frac{1 - \zeta_1^2\zeta_2^2 - (\zeta_1^2 - \zeta_2^2)\,\zeta_0}{(1-|\zeta_1|^2)(1-|\zeta_2|^2)(1-|\zeta_0|^2)^{1/2}} \\
&=\; \cosh(\tfrac{1}{\sqrt{2}}\lambda_0)\,\Big(\cosh^2(\tfrac{1}{2}\lambda_1)\cosh^2(\tfrac{1}{2}\lambda_2) - e^{-2i(\varphi_1-\varphi_2)}\sinh^2(\tfrac{1}{2}\lambda_1)\sinh^2(\tfrac{1}{2}\lambda_2)\Big) \\
&\qquad + e^{-i\varphi_0}\sinh(\tfrac{1}{\sqrt{2}}\lambda_0)\,\Big(\cosh^2(\tfrac{1}{2}\lambda_1)\sinh^2(\tfrac{1}{2}\lambda_2)e^{2i\varphi_2} - \sinh^2(\tfrac{1}{2}\lambda_1)\cosh^2(\tfrac{1}{2}\lambda_2)e^{-2i\varphi_1}\Big)\,.
\end{aligned} \tag{2.23}$$

Then the scalar potential is given by:

$$\begin{aligned}
\mathcal{P} \;&=\; 4g^2\sum_{j=0}^{2}\Big(\frac{\partial|\mathcal{W}|}{\partial\lambda_j}\Big)^2 \;+\; \frac{8\,g^2}{\sinh^2(\sqrt{2}\,\lambda_0)}\Big(\frac{\partial|\mathcal{W}|}{\partial\varphi_0}\Big)^2 \;+\; 4\,g^2\sum_{j=1}^{2}\frac{1}{\sinh^2(\lambda_j)}\Big(\frac{\partial|\mathcal{W}|}{\partial\varphi_j}\Big)^2 \;-\; 6\,g^2\,|\mathcal{W}|^2 \\
&=\; 8\,g^2\Big[(1-|\zeta_0|^2)^2\,\Big|\frac{\partial|\mathcal{W}|}{\partial\zeta_0}\Big|^2 \;+\; \frac{1}{2}\sum_{j=1}^{2}(1-|\zeta_j|^2)^2\,\Big|\frac{\partial|\mathcal{W}|}{\partial\zeta_j}\Big|^2 \;-\; 6\,|\mathcal{W}|^2\Big]\,.
\end{aligned} \tag{2.24}$$

One can also show that:

$$\mathcal{P} \;=\; 4\sum_{j=0}^{2}\Big|\frac{\partial\mathcal{W}}{\partial\lambda_j}\Big|^2 \;-\; 6\,|\mathcal{W}|^2\,, \tag{2.25}$$

which reflects the fact that $\mathcal{W}$ is holomorphic up to a scale factor.

As we will establish below, $|\mathcal{W}|$ defines a superpotential for the truncated theory and its critical points represent supersymmetric vacua. One can easily verify that the critical points of $|\mathcal{W}|$ are given by $\lambda_0 = \lambda_1 = \lambda_2 = 0$ (the $\mathcal{N}=8$ point) and by:

$$\begin{aligned} \tfrac{1}{\sqrt{2}}\lambda_0 \;&=\; \log\Big[\frac{1}{\sqrt{2}}\,(\sqrt{3}+1)\Big]\,, \qquad \lambda_1 \;=\; \lambda_2 = \log\big[\sqrt{3}\pm\sqrt{2}\big]\,, \\ \varphi_0 \;&=\; -\,\frac{\pi}{2}\,,\quad \varphi_1 \;=\; \varphi_2 \;=\; \frac{\pi}{4}\,;\quad \mathcal{P} \;=\; -\,12\,g^2\,. \end{aligned} \tag{2.26}$$

Note that $|\mathcal{W}|$ only has critical points with $\lambda_0 \geq 0$ whereas $\mathcal{P}$ has symmetric sets of critical points under $\lambda_0 \to -\lambda_0$ (see Fig. 1).

For future reference, it is convenient to introduce the phase, ω, of the complex superpotential:

$$\mathcal{W} \;=\; |\mathcal{W}|\,e^{i\omega}\,. \tag{2.27}$$

3 Supersymmetric flows

We take the four-dimensional metric to have the standard holographic flow form:

$$ds^2_{1,3} \;=\; dr^2 \;+\; e^{2A(r)}\big(\,\eta_{\mu\nu}\,dy^\mu\,dy^\nu\big) \tag{3.28}$$

and the Lagrangian of the scalars coupled to gravity is simply:

$$\mathcal{L} \;=\; \tfrac{1}{2}\,R \;-\; \mathcal{P} \;+\; \mathcal{L}_{kin.}\,. \tag{3.29}$$

From [29], the fermion supersymmetry variations, restricted to the scalar and gravitational sectors are:

$$\delta\psi^i_\mu \;=\; 2\,(\nabla_\mu\varepsilon^i \;+\; \tfrac{1}{2}\,\mathcal{B}_\mu{}^i{}_j\,\varepsilon^j) \;-\; \sqrt{2}\,g\,A_1^{ij}\gamma_\mu\varepsilon_j\,, \tag{3.30}$$

$$\delta\chi^{ijk} \;=\; -\mathcal{A}_\mu{}^{ijk\ell}\gamma^\mu\varepsilon_\ell \;-\; 2\,g\,A_2{}_\ell{}^{ijk}\varepsilon^\ell\,, \tag{3.31}$$

where we have explicitly written the scalar $SU(8)$ connection, $\mathcal{B}_\mu{}^i{}_j$.

As remarked earlier, the residual supersymmetry of the $\mathcal{N}=1$ sector is:

$$\eta \;\equiv\; \varepsilon^7+\varepsilon^8\,, \qquad \eta^* \;\equiv\; \varepsilon_7+\varepsilon_8\,, \tag{3.32}$$

where η is a Majorana spinor and η^* is its complex conjugate.

3.1 The supersymmetric flow equations

The supersymmetries along the flows may be written

$$\eta \;=\; e^{\frac{1}{2}A(r)}\, e^{\frac{1}{2}i\omega}\, \eta_0\,, \tag{3.33}$$

where $A(r)$ is the metric function in (3.28), ω is the phase of the complex superpotential defined in (2.27) and η_0 is a constant spinor. With this choice, the phase dependences cancel on both sides of (3.31) and one finds that theses supersymmetry variations imply:

$$\begin{aligned} \frac{d\lambda_j}{dr} \;&=\; \pm 2\sqrt{2}\, g\, \frac{\partial|\mathcal{W}|}{\partial\lambda_j}\,, \quad j=0,1,2\,; \qquad \frac{d\varphi_0}{dr} \;=\; \pm\, \frac{4\sqrt{2}\, g}{\sinh^2(\sqrt{2}\lambda_0)}\, \frac{\partial|\mathcal{W}|}{\partial\varphi_0}\,, \\ \frac{d\varphi_j}{dr} \;&=\; \pm \frac{2\sqrt{2}\, g}{\sinh^2(\lambda_j)}\, \frac{\partial|\mathcal{W}|}{\partial\varphi_j}\,, \quad j=1,2\,, \end{aligned} \tag{3.34}$$

where the signs are determined by the choice of the radial helicity projector on the supersymmetry:

$$\gamma^r\, \eta_0 \;=\; \pm\, \eta_0\,. \tag{3.35}$$

As usual, the complex superpotential is defined using the eigenvalues of the A_1-tensor:

$$A_1^{ij}\, v^j \;=\; \mathcal{W}\, v^j\,, \qquad \vec{v} \;\equiv\; (0,0,0,0,0,0,1,1)\,. \tag{3.36}$$

The gravitino variations in the y^μ directions of (3.28) then give the usual flow equation:

$$\frac{dA(r)}{dr} \;=\; \mp\, \sqrt{2}\, g\, |\mathcal{W}|\,. \tag{3.37}$$

The last supersymmetry condition is the radial component of the gravitino variation and this requires the non-trivial identity coming from the cancellation of the derivatives of the phase, ω, and terms arising from the $SU(8)$ connection, ${\mathcal{B}_\mu}^i{}_j$:

$$\frac{i}{2}\, \frac{d\omega}{dr} \;=\; i\, \sinh^2(\tfrac{1}{\sqrt{2}}\, \lambda_0)\, \frac{d\varphi_0}{dr} \;+\; 2\, i\, \sinh(\tfrac{1}{2}\, \lambda_1)\, \frac{d\varphi_1}{dr} \;-\; 2\, i\, \sinh(\tfrac{1}{2}\, \lambda_2)\, \frac{d\varphi_2}{dr}\,. \tag{3.38}$$

One can indeed verify that this equation is satisfied by virtue of the definition of ω in (2.27) and the flow equations (3.34).

Thus (3.34) and (3.37) generate families of supersymmetric flows in the $\mathcal{N}=1$ theory defined in Section 2. In particular, critical points of $|\mathcal{W}|$, like that given in (2.26), define supersymmetric vacua.

3.2 The new supersymmetric flow

We will choose the lower sign in (3.34) and (3.37) so that $A'(r) > 0$. One then finds that (3.34) implies

$$\frac{d|\mathcal{W}|}{dr} \;<\; 0\,. \tag{3.39}$$

along all flows away from critical points. Thus, as the c-theorem requires, $A'(r)$ and $|\mathcal{W}|$ increase as r decreases from infinity. One can easily check that

$$\frac{d|\mathcal{W}|}{d\varphi_0} \;=\; \frac{d|\mathcal{W}|}{d\varphi_1} \;=\; \frac{d|\mathcal{W}|}{d\varphi_2} \;=\; 0 \quad \text{for} \quad \varphi_0 = -\frac{\pi}{2}\,, \quad \varphi_1 = \varphi_2 = \frac{\pi}{4}\,, \tag{3.40}$$

but with *arbitrary* λ_j. One can also check that

$$\frac{d|\mathcal{W}|}{d\lambda_1} \;-\; \frac{d|\mathcal{W}|}{d\lambda_2} \;=\; 0 \quad \text{for} \quad \lambda_1 = \lambda_2\,, \quad \varphi_0 = -\frac{\pi}{2}\,, \quad \varphi_1 = \varphi_2\,. \tag{3.41}$$

In particular, it is consistent with the flow equations (3.34) to restrict to the space $\lambda_1 = \lambda_2$, $\varphi_0 = -\frac{\pi}{2}$, $\varphi_1 = \varphi_2 = \frac{\pi}{4}$ upon which one has a superpotential:

$$\mathcal{W} \;=\; \cosh(\lambda_1)\,\cosh(\tfrac{1}{\sqrt{2}}\,\lambda_0) \;-\; \frac{1}{2}\,\sinh^2(\lambda_1)\,\sinh(\tfrac{1}{\sqrt{2}}\,\lambda_0)\,. \tag{3.42}$$

This superpotential is plotted in the second graph in Fig. 1. One can easily see that there is a steepest descent solution to the flow between the $\mathcal{N}\!=\!8$ critical point and the new $\mathcal{N}\!=\!1$ critical point. We have solved this flow numerically and the result is plotted in the second graph in Fig. 1.

In the neighborhood of the $\mathcal{N}\!=\!8$ critical point the superpotential in (3.42) has the expansion

$$\mathcal{W} \;=\; 1 \;+\; \tfrac{1}{4}\,\lambda_0^2 \;+\; \tfrac{1}{2}\,\lambda_1^2 \;-\; \tfrac{1}{\sqrt{2}}\,\lambda_0\,\lambda_1^2 \;+\; \dots\,. \tag{3.43}$$

The flow equations are simply:

$$\frac{d\lambda_0}{dr} = -\frac{2}{L}\,\frac{\partial\mathcal{W}}{\partial\lambda_0} \approx -\frac{1}{L}\,\lambda_0\,, \qquad \frac{d\lambda_1}{dr} = -\frac{2}{L}\,\frac{\partial\mathcal{W}}{\partial\lambda_1} \approx -\frac{2}{L}\,\lambda_1\,, \qquad A'(r) = \frac{1}{L}\mathcal{W} \approx \frac{1}{L}\,, \tag{3.44}$$

where we have set $g = \frac{1}{\sqrt{2}L}$. Hence the flow starts out with:

$$\lambda_0 \;\approx\; a_0\, e^{-r/L}\,, \qquad \lambda_1 \;\approx\; a_1\, e^{-2r/L}\,, \qquad r \;\to\; \infty\,, \tag{3.45}$$

for some constants a_0 and a_1. This explains the nearly parabolic behavior of the flow near the origin in Fig. 1.

It is very interesting to note that λ_0 flows out along a *non-normalizable* mode in AdS_4 while λ_1 flows out along a *normalizable* mode in AdS_4. Also note that unlike the massive flows considered in [26], the leading UV behavior for λ_0 (parametrized by a_0) does not source the leading behavior in λ_1. These flows are thus independent in the UV except for the fine tuning required by targeting the new fixed point.

# of modes	m^2L^2	$SO(2)\times SO(2)$ irrep	$\mathcal{N}=1$ scalars
4	$2+\sqrt{15}+\sqrt{4+\sqrt{15}} \approx 8.679$	$(2,2)$	
1	$3(1+\sqrt{3}) \approx 8.196$	$(1,1)$	1
4	$\frac{5}{2}+\sqrt{10} \approx 4.412$	$(2,1)\oplus(1,2)$	
4	$2+\sqrt{15}-\sqrt{4+\sqrt{15}} \approx 3.067$	$(2,2)$	
1	$1+\sqrt{3} \approx 2.732$	$(1,1)$	1
30	0	$(2,2)_{\times 2}$ $(2,1)_{\times 4}\oplus(1,2)_{\times 4}$ $(1,1)_{\times 6}$	
1	$1-\sqrt{3} \approx -0.732$	$(1,1)$	1
4	$-\frac{5}{4}=-1.25$	$(2,1)\oplus(1,2)$	
4	$2-\sqrt{\frac{3}{2}}+\sqrt{\frac{5}{2}}-\sqrt{15} \approx -1.517$	$(2,2)$	
4	$\frac{5}{4}-\sqrt{10} \approx -1.912$	$(2,1)\oplus(1,2)$	
4	-2	$4\times(1,1)$	2
1	$3(1-\sqrt{3}) \approx -2.196$	$(1,1)$	1
4	$2-\sqrt{15}-\sqrt{4+\sqrt{15}} \approx -2.229$	$(2,2)$	
4	$-\frac{9}{4}=-2.25$	$(2,1)\oplus(1,2)$	

Table 1: The spectrum of scalars at the new $\mathcal{N}=1$ supersymmetric critical point.

3.3 The mass matrices at the new critical point.

The mass matrix for a general scalar fluctuation can be found in [30]. For a general scalar fluctuation, Σ_{ijkl}, one has, at quadratic order,

$$\begin{aligned}\mathcal{L}(\Sigma^2) = -\tfrac{1}{96}\, g^{\mu\nu}\partial_\mu\Sigma_{ijkl}\,\partial_\nu\Sigma^{ijkl} - \tfrac{g^2}{96}\Big(\big(\tfrac{2}{3}\,\mathcal{P}+\tfrac{13}{72}\,\big|A_{2\ell}{}^{ijk}\big|^2\big)\,\Sigma_{ijkl}\,\Sigma^{ijkl} \\ +\ \big(6\,A_{2k}{}^{mni}A_2{}^j{}_{mn\ell}\ -\ \tfrac{3}{2}\,A_{2n}{}^{mij}A_2{}^n{}_{mk\ell}\big)\,\Sigma_{ijpq}\,\Sigma^{klpq} \\ -\ \tfrac{2}{3}\,A_2{}^i{}_{mnp}A_{2q}{}^{jk\ell}\,\Sigma^{mnpq}\,\Sigma_{ijkl}\Big)\,.\end{aligned} \tag{3.46}$$

If one uses this formula at the $\mathcal{N}=1$ critical point one finds the scalar spectrum given in Table 1. The first column in the table contains the degeneracies, the second column has the masses and the third shows the charges under the residual symmetry. The last column shows the location of

six scalars that are in the $\mathcal{N}=1$ theory analyzed in Section 2.3. Those scalars are thus singlets under the symmetry that defined the truncation.

Note that all the masses obey the BF bound, as they must. It is interesting to observe that there are four fields that saturate the BF bound and there are four other fields whose dimension differs by one unit form these BF saturating fields. It seems likely that these fields form an $\mathcal{N}=1$ supermultiplet. It would also be interesting to see if these fields might be used for some form of Coulomb branch flow much like that investigated in [31, 32]. These fields are charged under the two residual $U(1)$'s and so such a flow would break all the symmetry.

4 The flow in the holographic field theory

In the neighborhood of the maximally supersymmetric point, the seventy scalars of the gauged supergravity theory are holographically dual to the (traceless) bilinears in the scalars and fermions:

$$\mathcal{O}^{IJ} \;=\; \mathrm{Tr}\,\big(X^I\,X^J\big) \;-\; \tfrac{1}{8}\,\delta^{IJ}\,\mathrm{Tr}\,\big(X^K\,X^K\big)\,, \quad I,J,\dots=1,\dots,8 \tag{4.47}$$

$$\mathcal{P}^{AB} \;=\; \mathrm{Tr}\,\big(\psi^A\,\psi^B\big) \;-\; \tfrac{1}{8}\,\delta^{AB}\,\mathrm{Tr}\,\big(\psi^C\,\psi^C\big)\,, \quad A,B,\dots=1,\dots,8\,, \tag{4.48}$$

where $\mathcal{O}^{IJ}$ transforms in the $\mathbf{35}_s$ of $SO(8)$, and $\mathcal{P}^{AB}$ transforms in the $\mathbf{35}_c$. The real parts of w_j in (2.4) can thus be thought of as the duals of some of the $\mathcal{O}^{IJ}$ and the imaginary parts of w_j can be thought of as the duals of some of the $\mathcal{P}^{IJ}$.

To make this map more specific, if one sets all of the angles $\varphi_j=0$, then the scalar fields of the $\mathcal{N}=1$ theory all lie in the $SL(8,\mathbb{R})$ subgroup of $E_{7(7)}$ and if one maps to this basis then one finds that the corresponding $SL(8,\mathbb{R})$ matrix may be written:

$$\mathcal{S} \;=\; \mathrm{diag}\big(e^{\mu_1}\,,e^{\mu_1}\,,e^{\mu_2}\,,e^{\mu_2}\,,e^{\mu_3}\,,e^{\mu_3}\,,e^{\mu_4}\,,e^{\mu_4}\big)\,, \tag{4.49}$$

$$\begin{aligned} \mu_1 \;&\equiv\; \frac{1}{2}\,\lambda_1-\frac{1}{2\sqrt{2}}\,\lambda_0\,, \qquad \mu_2 \;\equiv\; -\,\frac{1}{2}\,\lambda_1-\frac{1}{2\sqrt{2}}\,\lambda_0\,, \\ \mu_3 \;&\equiv\; \frac{1}{2}\,\lambda_2+\frac{1}{2\sqrt{2}}\,\lambda_0\,, \qquad \mu_4 \;\equiv\; -\,\frac{1}{2}\,\lambda_2+\frac{1}{2\sqrt{2}}\,\lambda_0\,. \end{aligned} \tag{4.50}$$

The fact that the critical flow has $\varphi_0=-\frac{\pi}{2}$ and $\varphi_1=\varphi_2=\frac{\pi}{4}$ means that the scalar field represented by w_0 lies entirely within the $\mathbf{35}_c$ sector while w_1 and w_2 are equally split between the two sectors. Moreover, we have $w_1=w_2$ and so the operators that are involved in the $\mathcal{N}=1$ flow described above may be written:

$$\mathcal{O}^{IJ} \;=\; \mathrm{diag}\big(e^{\nu_1}\,,e^{\nu_1}\,,e^{\nu_1}\,,e^{\nu_1}\,,e^{-\nu_1}\,,e^{-\nu_1}\,,e^{-\nu_1}\,,e^{-\nu_1}\big)\,, \tag{4.51}$$

$$\mathcal{P}^{AB} \;=\; \mathrm{diag}\big(e^{\nu_0+\nu_1}\,,e^{\nu_0+\nu_1}\,,e^{\nu_0-\nu_1}\,,e^{\nu_0-\nu_1}\,,e^{-\nu_0+\nu_1}\,,e^{-\nu_0+\nu_1}\,,e^{-\nu_0-\nu_1}\,,e^{-\nu_0-\nu_1}\big)\,, \tag{4.52}$$

$$\nu_0 \;\equiv\; \frac{1}{2\sqrt{2}}\,\lambda_0\,, \qquad \nu_1 \;\equiv\; \frac{1}{2\sqrt{2}}\,\lambda_1 \;=\; \frac{1}{2\sqrt{2}}\,\lambda_2\,. \tag{4.53}$$

# of modes	m^2L^2	$SO(3)\times SO(3)$ irrep	$\mathcal{N}=1$ scalars
1	$\frac{60}{7} \approx 8.571$	$(1,1)$	1
9	$\frac{3}{7}(5+\sqrt{65}) \approx 5.598$	$(3,3)$	1
9	$\frac{18}{7} \approx 2.571$	$(3,3)$	1
22	0	$(3,3)\oplus(3,1)_{\times 2}\oplus(1,3)_{\times 2}\oplus(1,1)$	
9	$\frac{3}{7}(5-\sqrt{65}) \approx -1.312$	$(3,3)$	1
20	$-\frac{12}{7} \approx -1.714$	$(3,3)_{\times 2}\oplus(1,1)_{\times 2}$	2

Table 2: The spectrum of scalars at the old $SO(3)\times SO(3)$-invariant critical point.

Non-normalizability of the λ_0 mode suggests that ν_0 is dual to a mass term while normalizability of the λ_1 mode suggests that ν_1 might be dual to a vacuum expectation value. Thus we appear to have a flow that involves an $SO(4)\times SO(4)$-invariant fermion mass term combined with some boson vevs and fermion condensates. If one sets $\nu_1 = 0$ and looks at the pure fermion mass flows then these preserve the sixteen supersymmetries of the $\mathcal{N}=4$ supergravity and indeed were studied extensively in [27, 28, 18]. In particular, such flows have infra-red fixed points that may be described, at least in the infra-red, in terms of free fermions in $(1+1)$-dimensions [18] and whose perturbations describe excitations of the Fermi sea. The new supersymmetric critical point involves adding some *normalizable fluxes* to this picture, which means that the flow involves a modification of the *state* of the holographic field theory and not a change of Lagrangian. Therefore, our new $\mathcal{N}=1$ flow has a potentially very interesting interpretation as a new (stable, supersymmetric) state of the fermion droplet theory and this state involves some form of bosonic vev and Fermi condensate.

It is also interesting to note that for this flow, the holographic central charge changes in a very simple way:

$$\frac{c_{\rm IR}}{c_{\rm UV}} \;=\; \left(\frac{\mathcal{P}_{\rm IR}}{\mathcal{P}_{\rm UV}}\right)^{-1} \;=\; \frac{1}{2}\,. \tag{4.54}$$

5 Stability of the $SO(3)\times SO(3)$-invariant point

If one uses (3.46) at the critical point with residual $SO(3)\times SO(3)$ symmetry and $\mathcal{P} = -14\,g^2$, one finds the scalar spectrum given in Table 2. Note that all the fields obey the BF bound and so this critical point is perturbatively stable.

The flow to this critical point cannot, of course, be supersymmetric, but it has already been discussed in [33]. Given the perturbative stability of this critical point, it would be well worth revisiting this flow to understand its role within AdS/CMT.

It is also important to note that the cosmological constant of this critical point, $-14g^2$, is lower than that of the new $\mathcal{N}=1$ point and, as is evident from the contours of the potential in Fig. 1, there must be a flow from the $\mathcal{N}=1$ point to the $SO(3)\times SO(3)$-invariant point. The relevant operator that drives this flow is dual to a combination of λ_0 and λ_1 and in the neighborhood of the $\mathcal{N}=1$ point this has $m^2L^2 = 3(1-\sqrt{3}) \approx -2.19615$. It would be most interesting to understand the role of this phase in the holographic field theory. Since the cosmological constants for the $SO(3)\times SO(3)$-point and the $\mathcal{N}=1$ point are integers, the ratios of central charges at the ends of all these flows are simple rational numbers, like $\frac{6}{7}$.

6 A new critical point with $SO(4)'$ invariance

The supersymmetric truncation in Section 2.3 was found initially using numerical results both for the continuous symmetry and exact location of the new $\mathcal{N}=1$ supersymmetric critical point (point #11 in [24] and Appendix B.1). We will now discuss an example where numerical constraints on the continuous symmetry and the mass spectrum are sufficient to set up a feasible analytic calculation of a new critical point.

A preliminary numerical search for critical points beyond the ones found in [24] (and listed in Appendix B.1) has identified a new critical point with a relatively large symmetry group. Indeed, numerical data for this point given in Appendix B.2 indicate six continous symmetries and a completely degenerate gravitino mass spectrum. This suggests that the symmetry group might be simply another $SO(3)\times SO(3)$, that is obtained by a triality rotation of the $SO(3)\times SO(3)$ in Section 5. To distinguish between the two we will denote this new symmetry by $SO(4)'$. It is explicitly realized by the following generators of $SO(8)$:

$$\begin{aligned}\alpha_i T_i^{(1)} + \beta_i T_i^{(2)} = {}& \alpha_1\, i\sigma^2\otimes\sigma^0\otimes\sigma^1 + \alpha_2\, i\sigma^0\otimes\sigma^0\otimes\sigma^2 + \alpha_3\, i\sigma^2\otimes\sigma^0\otimes\sigma^3 \\ & + \beta_1\, i\sigma^1\otimes\sigma^0\otimes\sigma^2 + \beta_2\, i\sigma^2\otimes\sigma^0\otimes\sigma^0 + \beta_3\, i\sigma^3\otimes\sigma^0\otimes\sigma^2\,,\end{aligned} \tag{6.55}$$

where σ^0 is the unit matrix and σ^i, $i=1,2,3$, are the Pauli matrices. Under these generators, the supersymmetries and the eight gravitini transform in two copies of the 4 of $SO(4)'$.

The eight components of a vector in the tensor product (6.55) are ordered as $(111), (112), (121)$, etc., and correspond to the eight vector components of $SO(8)$. There are

six invariant, noncompact generators of $E_{7(7)}$ spanned by the following forms:

$$\begin{aligned}\Psi_0 = w_0 \Big[& dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8 \\ & - dx^1 \wedge dx^3 \wedge dx^5 \wedge dx^7 - dx^2 \wedge dx^4 \wedge dx^6 \wedge dx^8 \\ & - dx^1 \wedge dx^3 \wedge dx^6 \wedge dx^8 - dx^2 \wedge dx^4 \wedge dx^5 \wedge dx^7 \Big] ,\end{aligned} \tag{6.56}$$

$$\begin{aligned}\Psi_1 = w_1 \Big[& dx^1 \wedge dx^2 \wedge dx^7 \wedge dx^8 + dx^3 \wedge dx^4 \wedge dx^5 \wedge dx^6 \\ & + dx^1 \wedge dx^4 \wedge dx^5 \wedge dx^8 + dx^2 \wedge dx^3 \wedge dx^6 \wedge dx^7 \\ & - dx^1 \wedge dx^4 \wedge dx^6 \wedge dx^7 - dx^2 \wedge dx^3 \wedge dx^5 \wedge dx^8 \Big] ,\end{aligned} \tag{6.57}$$

$$\begin{aligned}\Psi_2 = w_2 \Big[& dx^1 \wedge dx^2 \wedge dx^5 \wedge dx^8 - dx^1 \wedge dx^2 \wedge dx^6 \wedge dx^7 \\ & + dx^1 \wedge dx^4 \wedge dx^5 \wedge dx^6 - dx^2 \wedge dx^3 \wedge dx^5 \wedge dx^6 \Big] \\ -\overline{w}_2 \Big[& dx^1 \wedge dx^4 \wedge dx^7 \wedge dx^8 - dx^2 \wedge dx^3 \wedge dx^7 \wedge dx^8 \\ & + dx^3 \wedge dx^4 \wedge dx^5 \wedge dx^8 - dx^3 \wedge dx^4 \wedge dx^6 \wedge dx^7 \Big] ,\end{aligned} \tag{6.58}$$

$$\Psi_3 = w_3 \, dx^1 \wedge dx^2 \wedge dx^5 \wedge dx^6 + \overline{w}_3 \, dx^3 \wedge dx^4 \wedge dx^7 \wedge dx^8 \, , \tag{6.59}$$

where

$$w_0 = \frac{1}{\sqrt{3}}\lambda_0 \, , \qquad w_1 = \frac{1}{\sqrt{3}}\lambda_1 \, , \qquad w_2 = -\frac{1}{2}(\lambda_2 - i\lambda_4) \, , \qquad w_3 = \lambda_3 + i\lambda_5 \, . \tag{6.60}$$

The real parameters λ_i, $i = 0, \ldots, 5$, correspond to canonically normalized generators and parametrize the coset space:

$$O(1,1) \times \frac{SL(3,\mathbb{R})}{SO(3)} \, , \tag{6.61}$$

where λ_0 is the coordinate on the first factor, while $\lambda_1, \ldots, \lambda_5$ are coordinates on the second factor with the corresponding noncompact generators of $SL(3,\mathbb{R})$ given by

$$\Lambda = \begin{pmatrix} \frac{2}{\sqrt{3}}\lambda_1 & \lambda_2 & \lambda_4 \\ \lambda_2 & -\frac{1}{\sqrt{3}}\lambda_1 + \lambda_3 & \lambda_5 \\ \lambda_4 & \lambda_5 & -\frac{1}{\sqrt{3}}\lambda_1 - \lambda_3 \end{pmatrix} . \tag{6.62}$$

The unbroken $O(2)$ gauge symmetry acts on Λ as the $\mathcal{R}_{12}$ rotation and allows one to set a linear combination of λ_4 and λ_5 to zero. This reduces the number of independent parameters in the potential to five.

To calculate the potential, one must exponentiate the $E_{7(7)}$ generators Ψ_0 and $\Psi = \Psi_1 + \Psi_2 + \Psi_3$. While this is completely straightforward for the first generator, it is extremely difficult for

the second one if one insists on keeping $\lambda_1, \ldots, \lambda_5$ as explicit parameters. Instead, we express the potential as a function of the matrix elements, m_{ij}, of the $SL(3,\mathbb{R})$ group element

$$M = e^{\Lambda}\,, \qquad M = (m_{ij})_{i,j=1,2,3}\,. \tag{6.63}$$

Note that by construction M is symmetric, $m_{ij} = m_{ji}$. The exponentiation is accomplished by a similarity transformation, S, that brings Ψ to the block diagonal form,

$$S^{-1}\Psi S = \mathrm{diag}(6\times(\Lambda,-\Lambda), 20\times 0)\,. \tag{6.64}$$

The potential is a sixth-order polynomial in $\rho = e^{-2\lambda_0/\sqrt{3}}$ and the m_{ij}'s. It becomes algebraically quite manageable if we use the remaining gauge symmetry to set a linear combination of the matrix elements m_{13} and m_{23} to zero. It can then be further simplified by solving explicitly the unimodularity condition, $\det M = 1$, to eliminate one additional matrix element. The final result is given in Appendix A.

It is clear that a systematic search for critical points of the full potential (A.2) is still quite involved, and we have not carried it out in detail. In the following we consider two natural restrictions in which we keep only two commuting fields: the $O(1,1)$ field, λ_0, and one additional field, λ, in $SL(3,\mathbb{R})$.

First, take Λ, and hence M, to be diagonal by setting

$$\lambda_1 = \frac{\sqrt{3}}{2}\,\lambda\,, \qquad \lambda_3 = -\frac{1}{2}\,\lambda\,, \qquad \lambda_2 = \lambda_4 = \lambda_5 = 0\,. \tag{6.65}$$

The restriction of the potential (A.2) to λ_0 and λ is

$$\mathcal{P} = -g^2\left[3\,\rho^{-1} + 3\,\rho\,\cosh(2\lambda) + \frac{1}{4}\,\rho^3(1-\cosh(4\lambda))\right], \qquad \rho = e^{-2\lambda_0/\sqrt{3}}\,, \tag{6.66}$$

and, apart from the trivial point $\lambda_0 = \lambda = 0$, there is one nontrivial critical point at

$$\lambda_0 = -\frac{\sqrt{3}}{8}\,\log(5)\,, \qquad \lambda = \pm\frac{1}{4}\,\log(5)\,, \qquad \mathcal{P} = -2\cdot 5^{3/4}\,g^2\,. \tag{6.67}$$

The value of the cosmological constant identifies this point as the $SO(7)^+$ critical point. By expanding (6.66) to the quadratic order and using that λ_0 and λ have canonical kinetic terms, we find two masses: $m^2L^2 = 6$ and $-12/5$. Those are indeed correct values for the masses of scalar fluctuations at this point [30] (see, also [13] and Table B.1), which suggests that (6.65) is a consistent truncation in this sector. Furthermore, we see that there is one unstable mode, which arises as a linear combination of the two modes in (6.65).

The second natural restriction is with completely off-diagonal Λ. It is clear from the form of the generators Ψ_i in (6.57)-(6.59) that the simplest choice is to take

# of modes	m^2L^2	$SO(4)' \simeq SO(3) \times SO(3)$ irrep
1	$3+\sqrt{3}+\sqrt{6(4+\sqrt{3})} \;\approx\; 10.597$	$(1,1)$
1	$\frac{3}{2}(5+\sqrt{3}) \;\approx\; 10.098$	$(1,1)$
1	$4\sqrt{3} \;\approx\; 6.928$	$(1,1)$
9	$2(\sqrt{3}-1) \;\approx\; 1.464$	$(3,3)$
22	0	$(3,3)\oplus(3,1)_{\times 2}\oplus(1,3)_{\times 2}\oplus(1,1)$
9	$2(\sqrt{3}-2) \;\approx\; -0.536$	$(3,3)$
1	$3+\sqrt{3}-\sqrt{6(4+\sqrt{3})} \;\approx\; -1.132$	$(1,1)$
15	$\frac{3}{2}(\sqrt{3}-3) \;\approx\; -1.902$	$(3,3)\oplus(3,1)\oplus(1,3)$
10	$2(\sqrt{3}-3) \;\approx\; -2.536$	$(5,1)\oplus(1,5)$
1	$-2\sqrt{3} \;\approx\; -3.464$	$(1,1)$

Table 3: The spectrum of scalars at the $SO(4)'$ point.

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0\,, \qquad \lambda_5 = \lambda\,. \tag{6.68}$$

This leads to the potential

$$\mathcal{P} = -\frac{g^2}{4}\,\rho^{-1}\left[\,(\rho^4 - 6\rho^2 - 3)\,\cosh(2\lambda) - (\rho^2+3)^2\,\right]\,, \qquad \rho = e^{-2\lambda_0/\sqrt{3}}\,. \tag{6.69}$$

A straightforward calculation reveals one nontrivial critical point at

$$\lambda_0 = -\frac{\sqrt{3}}{4}\,\log(3+2\sqrt{3})\,, \qquad \lambda = \pm\frac{1}{2}\,\mathrm{arccosh}(\sqrt{3})\,, \tag{6.70}$$

with the cosmological constant

$$\mathcal{P} = -6\,\sqrt{1+\frac{2}{\sqrt{3}}}\,g^2 \;\approx\; -8.807\,g^2\,. \tag{6.71}$$

Indeed, this is the same value as found in the numerical search in Appendix B.2. The calculation of the scalar masses is summarized in Table 3 and agrees with the numerical result. The two masses on the restricted fields are $m^2L^2 = 3\sqrt{3}$ and $-2\sqrt{3} \approx -3.464$, which once more suggests a consistent truncation while the latter exhibits one of the unstable modes at this point. Other unstable modes transform in $(5,1)\oplus(1,5)$ of $SO(4)'$ and hence are not visible in this truncation.

7 The new critical points with at most $U(1)^2$ symmetry

Given that the $SO(8)$ gauged $\mathcal{N}=8$ model indeed *does* have non-supersymmetric perturbatively stable vacua, an obvious question is how many there are and, given that the total number of critical points may well be very large, whether or not stability without supersymmetry is a rare.

Using the numerical data provided in [25] to study the stability of the fourteen new critical points that were presented in [24] to determine scalar masses, one finds that the thirteen solutions without residual supersymmetry all violate the BF bound $m^2_{\mathrm{scalar}}L^2 \geq -9/4 = -2.25$ so strongly that this cannot be attributed to the limited numerical accuracy to which their positions have been determined. Details on the scalar mass matrix eigenvalues and their degeneracies are given in Appendix B.1. This seems to suggest that stability without supersymmetry indeed is a rare phenomenon in four dimensions.[2] It is almost a comical coincidence that the very first critical point that was found through a systematic analysis of the scalar potential [19] is, at present, the only known one that is non-supersymmetric and stable!

7.1 Genericity of the new critical points

Another important question about all these new critical points is how generic they are and to what extent to which they represent an exhaustive list or whether they are a "random sample" of perhaps many undiscovered points.

There are two important aspects to this issue. First, the numerical method uses the sensitivity back-propagation method to minimize $|Q|^2$, where Q_{ijkl} is the tensor self-duality condition that defines a critical point [30, 24]. The algorithm starts from a choice of a random point but even so, it provides a remarkably efficient strategy to numerically obtain a lot of useful information about previously unknown stationary points which may then be utilized as an input to a fully analytic investigation. However, the size of the basin of attraction seems to vary strongly between different stationary points. Numerical searches can have four different outcomes: (a) the calculation fails due to a numerical overflow, (b) the calculation 'gets stuck' as optimization proceeds extremely slowly, (c) the calculation produces a stationary point which already was found earlier, and (d) the calculation produces a novel solution. With some code tweaking, (a) can be avoided almost completely, (b) happens rarely, but quite typically in the neighborhood of some 'tricky' critical points (the $\mathcal{N}=1$ vacuum with $\mathcal{P}/g^2 = -12$ being one of them), (c) is fairly frequent but with a very uneven distribution of results, and (d) happens sufficiently often to assume that the total number of solutions is far larger than those described so far.

The second aspect is that typical violations of the stationarity condition, measured by $|Q|^2$,

[2]In three dimensions the situation is almost the opposite: One finds that most of the three-dimensional, non-supersymmetric critical points are BF-stable.

get larger the further one moves outward from the $\mathcal{N}=8$ vacuum on the scalar manifold. Hence, gradients naturally tend to draw the numerical optimization towards the $\mathcal{N}=8$ point. In order to counteract this, the numerical search strategy which was developed in [34] for $\mathcal{N}=16$ theory in three dimensions and also employed to find the fourteen new solutions comes with a 'tweaking parameter' that essentially allows some control over the distance from the $\mathcal{N}=8$ point at which optimization will tend to spend most time. In [24], this parameter was chosen to scan at a distance somewhat beyond the $SU(3)$-invariant critical points. Due to the existence of this tweaking parameter, all claims about how generic the solutions thus found by this approach must be qualified with a statement about the search range.

While a far more detailed analysis of the scalar potentials of all (highly extended) supergravity models is now possible compared to what would have been considered as feasible some years ago, there also are some indications (considering results for the $\mathcal{N}=16$ theory in three dimensions [34]) that hardware-supported IEEE-754 floating point numbers might be insufficient to find and analyze solutions that lie 'very far out'. Apart from this, high-precision numerics also is important for semi-automatically producing analytic conjectures from numerical data, as explained in [25]. This will be explored in more detail in the next sub-sections.

One generally would expect that analytic expressions for the locations and properties of critical points get ever more complicated the more the $SO(8)$ gauge symmetry is broken. As explained in [25], the large number of solutions suggests using semi-automatic heuristics to obtain analytic expressions from numerical data. We will now illustrate this approach by proposing some analytic results for critical point #9 in the list presented in [24]. This solution has residual $U(1)\times U(1)$ gauge symmetry but as we will see, the analytic form of the solution is very complicated. We then discuss critical point #8, which has *no* residual continuous symmetry, and argue that it is, at present, completely out of reach of analytic methods.

7.2 Critical point #9

Using the sensitivity back-propagation method with enhanced numerical precision allows the determination of the cosmological constant of critical point #9 to high accuracy with reasonable computational effort. To 100 digits, this is:

$$\begin{aligned}\mathcal{P}^{\#9}/g^2 \;\approx\; & -10.674754182994893767797976935958061653722260124560 5\\ & 8812167158899484460464464439307664949292077482553.\end{aligned} \tag{7.72}$$

Employing the PSLQ algorithm [35] to find integer relations between integer powers of $\mathcal{P}^{\#9}/g^2$, one finds that about 80 digits of the cosmological constant suffice to automatically derive the conjecture that $\left(\mathcal{P}^{\#9}/g^2\right)^4$ is a zero of the polynomial

$$x \mapsto -27x^3 + 351\,632x^2 - 13\,574\,400x + 405\,504. \tag{7.73}$$

Specifically, if we define:

$$\begin{aligned} Q &= 6561 \\ R &= 28482192 \\ S &= 122545537024 \\ W &= ((128692865796145152 + 20596696547328\sqrt{2343}\,i)/1594323)^{1/3}\,, \end{aligned} \tag{7.74}$$

then we find:

$$V^{\#9}/g^2 \;=\; \left(\frac{QW^2 + RW + S}{QW}\right)^{1/4} \tag{7.75}$$

It is also interesting to note that there is no natural number $N < 100000$ for which $(W/|W|)^N = 1$.

This exact expression then manages to 'predict' the next 20 digits correctly and hence very likely is the correct one. The complexity of the analytic expression shown here has to be seen in contrast to those that give the cosmological constant for the long-known critical points #1 – #7: There, the fourth power of $-\mathcal{P}/g^2$ always is a fairly manageable rational number.

Given that $\mathcal{P}^{\#9}/g^2$ is a zero of a 12^{th} order polynomial with coefficients of magnitude $< 10^9$, one may optimistically hope that a reasonable number of digits of accuracy should suffice to automatically determine similar analytic conjectures for the entries of the $E_{7(7)}$ 56−bein $\mathcal{V}$. However, as table (A.1) in [13] shows, the coordinates of stationary points generally can be expected to be algebraically somewhat more complicated than their cosmological constants, and this seems to hold as well for the entries of the 56-bein. So far, 150 digits of accuracy turned out to be insufficient to obtain any analytic conjectures for this critical point by applying the PSLQ algorithm. Still, once analytic expressions for the entries of $\mathcal{V}$ are known, exact analytic results for all other properties can be derived automatically. In particular, the $Q^{ijkl} = 0$ stationarity condition then can be checked analytically. The same claims also hold for other stationary points and for models in other dimensions, however, taking the logarithm of $\mathcal{V}$ and also establishing that $\mathcal{V}$ is indeed an element of the corresponding exceptional group may sometimes be somewhat tricky (especially in three dimensions). For critical point #9, the 56-bein $\mathcal{V}$ contains (in its real and imaginary part) 92 different irrational numbers for which analytic expressions yet have to be found. Nevertheless, the observation that the stationarity condition could be satisfied to more than 100 digits leaves little room for doubt about the existence of this particular solution.

Precision data on the location of solution #9 are listed in Appendix B.3.

7.3 Critical point #8

The hybrid analytic/numerical heuristics based on the PSLQ algorithm can be applied to all the other critical points but, at present, this algorithm does not appear to be strong enough to obtain any analytic conjecture for properties of the least symmetric points. For example,

at critical point #8 in [24] the gauge group is broken completely and this point illustrates the magnitude of the analytic challenge.

Among those critical points without residual symmetry, one would naturally expect the one with smallest $-\mathcal{P}/g^2$ to have the simplest analytic expressions. The value of the cosmological constant (accurate to 300 digits) is:

$$\begin{aligned}\mathcal{P}^{\#8}/g^2 \quad \approx \quad & -10.4347125950092267924281315075560480704656700843520 \\ & 0322311834318562290776959926782172785944783577840 \\ & 6623358857637846084918638559407725956186947264350 \\ & 6529246437162270136399503087625029060959140682610 \\ & 2158309003737688359389115105789592868640965012710 \\ & 0298846950346167151558184534276762707591471282900 \\ & 476481455688\end{aligned} \tag{7.76}$$

This information should be sufficient to obtain a useful analytic conjecture for $\mathcal{P}/g^2$ if it is the zero of a polynomial with integer coefficients such that

$$\langle\text{degree}\rangle \cdot \langle\text{number of digits in max coeff}\rangle < 300 - n, \quad n \approx 20.$$

So far, we have not managed to obtain such an expression, indicating that the complexity of the corresponding polynomial is beyond reach even of this level of accuracy. We give precision data on the location of critical point #8 to 150 digits in Appendix B.3 and leave it as a challenge to our readers to obtain an analytic result.

This investigation shows that, in all likelihood, analytic expressions for further critical points (which are very likely to exist) with no or very little residual symmetry and large $\mathcal{P}/\mathcal{P}_0$ probably are too complicated to be handled conveniently. Hence, using numerical methods may well be the dominant strategy to obtain information about the properties of most as yet unknown critical points.

8 Conclusions

It is evident that, even many years after its construction, the potential of gauged $\mathcal{N}=8$ supergravity in four dimensions still has the capacity to surprise us with new challenges and potentially physically important solutions.

In this paper we have not only analytically exhibited a new, $\mathcal{N}=1$ supersymmetric critical point but we have embedded it in a consistent truncation to an $\mathcal{N}=1$ supersymmetric field theory. We have given the complete superpotential for this theory and have derived the supersymmetric flow equations and used them to construct the flow from the $\mathcal{N}=8$ supersymmetric critical point

to the new $\mathcal{N}=1$ point. The holographic interpretation of this flow is potentially very interesting because it appears to correspond to a new superconformal state in a broad class of flows whose original infra-red limit corresponds to a free fermion model [18].

We have also done some analysis of the new families of solutions that have been found numerically [24, 25]. In addition to the discovery of the new $\mathcal{N}=1$ supersymmetric point, the numerical search also revealed thirteen new non-supersymmetric critical points. Unfortunately, all of these new points have scalar excitations that violate the BF bound and so are unstable. On the other hand, our complete stability analysis revealed that the $SO(3)\times SO(3)$-invariant critical point, found long ago, is perturbatively stable. It is also a critical point that lies in the truncation to the $\mathcal{N}=1$ supergravity theory and so it is easily analyzed from within a simple and very interesting supergravity theory. It is also intriguing to note that this $\mathcal{N}=1$ supergravity has the somewhat unusual property that its potential has several critical points and *all* of them are stable! This could possibly be related to its interesting holographic dual.

One of the other things that is clear from the numerical analysis is that there are probably a lot more critical points in the supergravity potential. Moreover, a lot of them have little, or no, residual symmetry. We have demonstrated how such critical points with low levels of symmetry are going to be very hard, if not impossible, to access analytically. On the other hand, the best strategy is probably the one exemplified by this paper: Use the numerical algorithms to find the interesting, stable points and then, once one knows where to look, bring the full force of analytic methods to bear on the new solutions.

As regards the future, there is evidently a lot of new critical points to be discovered and one should also look in the five-dimensional maximal supergravity theories as well. From the point of view of AdS/CMT, there are also a number of interesting new things to be done.

First, as we pointed out in Section 2, the $\mathcal{N}=1$ supergravity that contains the new, $\mathcal{N}=1$ supersymmetric critical point can itself be embedded in slightly larger $\mathcal{N}=2$ supergravity theories that contain some vector potentials. These vector potentials can be used to induce chemical potentials in the theory on the brane and so drive interesting flows to the critical points of the theory, particularly given the stability of all the critical points in the $\mathcal{N}=1$ theory. This would be especially interesting given the well-known holographic interpretation of the massive flows within this model.

It is also important to point out that the massive flows with sixteen supersymmetries in gauged supergravity studied in [27] represented an extremely simple "tip of an iceberg" when the corresponding flows were studied in much greater generality in M-theory and IIB supergravity [28, 18]. Since the flows studied here differ from the flows of [27] by adding perturbations by normalizable modes, it is reasonable to expect that something similar might occur in the far more general flows constructed in [28, 18]. Thus it would be extremely interesting to lift the solutions

considered here to solutions of M-theory. On the face of it, this is a dauntingly difficult problem because of the lack of symmetry: Only a $U(1) \times U(1)$. On the other hand, there might be other aspects to the geometry that could make the computations feasible, particularly if the underlying manifolds were Sasaki-Einstein, or something similar. As yet, we have no basis for belief, one way or the other, in such a geometric simplification, but we think that this new supersymmetric phase is sufficiently interesting that it warrants a great deal of effort in trying to characterize its underlying geometry in as simple and universal manner as possible.

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A The scalar potential in the $SO(4)'$ sector

The scalar potential in the $SO(4)'$ sector is a function of an $O(1,1)$ group element, $\rho = e^{-2\lambda_0/\sqrt{3}}$, and a symmetric $SL(3,\mathbb{R})$ matrix, $M = (m_{ij})$. We fix the gauge by setting $m_{23} = 0$ and explicitly solve the constraint, $\det M = 1$,

$$m_{33} = \frac{1}{\Delta}(1 + m_{13}^2 m_{22}^2)\,, \qquad \Delta = m_{11}m_{22} - m_{12}^2\,, \tag{A.1}$$

thereby eliminating m_{33}. The resulting potential depends on five parameters: ρ, m_{11}, m_{12}, m_{22} and m_{13}, and is given by

$$\mathcal{P} = \frac{g^2}{8\Delta^2}\,\left[\,3\rho^{-1}P_{-1} + 6\rho\,P_1 + \rho^3 P_3\,\right]\,, \tag{A.2}$$

where

$$\begin{aligned} P_{-1} = &- 1 - 6m_{12}^4 - m_{12}^8 - m_{12}^4 m_{13}^2 - m_{12}^6 m_{13}^2 + 12 m_{11} m_{12}^2 m_{22} + 4 m_{11} m_{12}^6 m_{22} \\ &- 2m_{13}^2 m_{22} + 2m_{11}m_{12}^2 m_{13}^2 m_{22} + 2m_{11} m_{12}^4 m_{13}^2 m_{22} - 6m_{11}^2 m_{22}^2 - 6m_{11}^2 m_{12}^4 m_{22}^2 \\ &- m_{11}^2 m_{13}^2 m_{22}^2 - m_{11}^2 m_{12}^2 m_{13}^2 m_{22}^2 - m_{12}^4 m_{13}^2 m_{22}^2 - m_{13}^4 m_{22}^2 + 4m_{11}^3 m_{12}^2 m_{22}^3 \\ &+ 2m_{11} m_{12}^2 m_{13}^2 m_{22}^3 - m_{11}^4 m_{22}^4 - m_{11}^2 m_{13}^2 m_{22}^4\,, \end{aligned} \tag{A.3}$$

$$\begin{aligned} P_1 = &- m_{11}^2 - 2m_{12}^2 - m_{11}^2 m_{12}^4 - 2m_{12}^6 - 2m_{11}m_{12}^2 m_{13}^2 - m_{12}^4 m_{13}^2 - m_{12}^6 m_{13}^2 \\ &- m_{12}^4 m_{13}^4 + 2m_{11}^3 m_{12}^2 m_{22} + 4m_{11} m_{12}^4 m_{22} - 4m_{12}^2 m_{13}^2 m_{22} + 2m_{11} m_{12}^2 m_{13}^2 m_{22} \\ &+ 2m_{11} m_{12}^4 m_{13}^2 m_{22} - m_{22}^2 - m_{11}^4 m_{22}^2 - 2m_{11}^2 m_{12}^2 m_{22}^2 - m_{12}^4 m_{22}^2 - m_{11}^2 m_{13}^2 m_{22}^2 \\ &- m_{11}^2 m_{12}^2 m_{13}^2 m_{22}^2 - m_{12}^4 m_{13}^2 m_{22}^2 - 2m_{12}^2 m_{13}^4 m_{22}^2 + 2m_{11} m_{12}^2 m_{22}^3 - 2m_{13}^2 m_{22}^3 \\ &+ 2m_{11} m_{12}^2 m_{13}^2 m_{22}^3 - m_{11}^2 m_{22}^4 - m_{11}^2 m_{13}^2 m_{22}^4 - m_{13}^4 m_{22}^4\,, \end{aligned} \tag{A.4}$$

$$\begin{aligned} P_3 = \;& m_{11}^4 + 4m_{11}^2 m_{12}^2 + m_{11}^2 m_{13}^2 + m_{12}^2 m_{13}^2 + 2m_{11}^3 m_{12}^2 m_{13}^2 + 4m_{11} m_{12}^4 m_{13}^2 \\ &+ m_{11}^2 m_{12}^6 m_{13}^2 + m_{12}^8 m_{13}^2 + 2m_{11} m_{12}^2 m_{13}^4 + m_{11}^2 m_{12}^4 m_{13}^4 + 2m_{12}^6 m_{13}^4 \\ &+ m_{12}^4 m_{13}^6 + 8m_{11} m_{12}^2 m_{22} + 4m_{11}^2 m_{12}^2 m_{13}^2 m_{22} + 6m_{12}^4 m_{13}^2 m_{22} \\ &- 2m_{11}^3 m_{12}^4 m_{13}^2 m_{22} + 2m_{12}^2 m_{13}^4 m_{22} - 2m_{11}^2 m_{22}^2 + 4m_{12}^2 m_{22}^2 \\ &+ 6m_{11} m_{12}^2 m_{13}^2 m_{22}^2 + m_{11}^4 m_{12}^2 m_{13}^2 m_{22}^2 - 3m_{11}^2 m_{12}^4 m_{13}^2 m_{22}^2 + 3m_{12}^6 m_{13}^2 m_{22}^2 \\ &+ 2m_{11}^2 m_{12}^2 m_{13}^4 m_{22}^2 + 4m_{12}^4 m_{13}^4 m_{22}^2 + m_{12}^2 m_{13}^6 m_{22}^2 + 8m_{12}^2 m_{13}^2 m_{22}^3 \\ &+ 2m_{11}^3 m_{12}^2 m_{13}^2 m_{22}^3 - 6m_{11} m_{12}^4 m_{13}^2 m_{22}^3 + 2m_{11} m_{12}^2 m_{13}^4 m_{22}^3 + m_{22}^4 \\ &+ 3m_{11}^2 m_{12}^2 m_{13}^2 m_{22}^4 + m_{12}^4 m_{13}^2 m_{22}^4 + 4m_{12}^2 m_{13}^4 m_{22}^4 + 2m_{13}^2 m_{22}^5 \\ &- 2m_{11} m_{12}^2 m_{13}^2 m_{22}^5 + m_{11}^2 m_{13}^2 m_{22}^6 + m_{13}^4 m_{22}^6\,. \end{aligned} \tag{A.5}$$

B Numerical data

B.1 Mass matrices

For convenience, numerical data on scalar masses are listed in table B.1. These values have been obtained by (3.46) and verified independently via taking second order derivatives numerically. The labeling of stationary points parallels [24], while the AdS mass scale conventions match those used in [13]. As the actual position of most new critical points is only known to limited accuracy, it is conceivable that better location data would in some cases give very slightly different masses. Still, these numerical data clearly show that only one of the 17 non-supersymmetric critical points that are now known is perturbatively stable.

#0: $\mathcal{P}/g^2 = -6.000000$, $\mathcal{N} = 8$	
$m^2_{\rm scalar}L^2$	`-2.000`$_{\times 70}$
#1: $\mathcal{P}/g^2 = -6.687403$, $SO(7)_+$	
$m^2_{\rm scalar}L^2$	`-2.400`$_{\times 27}$, `-1.200`$_{\times 35}$, `0.000`$_{\times 7}$, `6.000`
#2: $\mathcal{P}/g^2 = -6.987712$, $SO(7)_-$	
$m^2_{\rm scalar}L^2$	`-2.400`$_{\times 27}$, `-1.200`$_{\times 35}$, `0.000`$_{\times 7}$, `6.000`
#3: $\mathcal{P}/g^2 = -7.191576$, G_2 $\mathcal{N} = 1$	
$m^2_{\rm scalar}L^2$	`-2.242`$_{\times 27}$, `-1.425`$_{\times 27}$, `0.000`$_{\times 14}$, `1.550`, `6.449`
#4: $\mathcal{P}/g^2 = -7.794229$, $SU(3) \times U(1)$ $\mathcal{N} = 2$	
$m^2_{\rm scalar}L^2$	`-2.222`$_{\times 12}$, `-2.000`$_{\times 16}$, `-1.556`$_{\times 18}$, `-1.123`, `0.000`$_{\times 19}$, `2.000`$_{\times 3}$, `7.123`
#5: $\mathcal{P}/g^2 = -8.000000$, $SU(4)_-$	
$m^2_{\rm scalar}L^2$	`-3.000`$_{\times 20}$, `-0.750`$_{\times 20}$, `0.000`$_{\times 28}$, `6.000`$_{\times 2}$
#6: $\mathcal{P}/g^2 = -14.000000$, $SO(3) \times SO(3)$	
$m^2_{\rm scalar}L^2$	`-1.714`$_{\times 20}$, `-1.312`$_{\times 9}$, `0.000`$_{\times 22}$, `2.571`$_{\times 9}$, `5.598`$_{\times 9}$, `8.571`$_{\times 1}$
#7: $\mathcal{P}/g^2 = -9.987083$, $U(1)$	
$m^2_{\rm scalar}L^2$	`-3.051`, `-2.476`, `-2.433`$_{\times 2}$, `-2.197`$_{\times 2}$, `-2.094`, `-1.968`$_{\times 2}$, `-1.815`$_{\times 2}$, `-1.814`$_{\times 2}$, `-1.791`$_{\times 2}$, `-1.768`$_{\times 2}$, `-1.764`$_{\times 2}$, `-1.606`, `-1.398`$_{\times 2}$, `-1.349`, `-1.332`$_{\times 2}$, `-1.330`$_{\times 2}$, `-1.279`$_{\times 2}$, `-1.268`$_{\times 2}$, `-1.007`, `-0.662`$_{\times 2}$, `0.000`$_{\times 27}$, `1.732`, `4.520`, `4.884`$_{\times 2}$, `5.576`$_{\times 2}$, `7.355`, `7.486`$_{\times 2}$
#8: $\mathcal{P}/g^2 = -10.434713$, $\langle$No gauge symmetry$\rangle$	

$m^2_{\rm scalar}L^2$	`-3.076, -2.598, -2.568,` $-2.451_{\times 2}$`, -2.407, -2.349, -2.118, -1.874, -1.863,` $-1.847_{\times 2}$`, -1.826,` $-1.792_{\times 2}$`,` $-1.736_{\times 2}$`, -1.699, -1.691,` $-1.526_{\times 2}$`, -1.377, -1.352,` $-1.189_{\times 2}$`, -1.166, -1.158, -1.027, -0.971, -0.640,` $0.000_{\times 28}$`, 0.496, 1.295, 3.516, 4.074,` $4.205_{\times 2}$`, 5.378,` $6.156_{\times 2}$`, 7.380, 7.462, 7.937`

#9: $\mathcal{P}/g^2 = -10.674754$, $U(1) \times U(1)$	
$m^2_{\rm scalar}L^2$	`-3.367,` $-3.292_{\times 2}$`, -2.638, -2.472,` $-2.132_{\times 2}$`,` $-2.086_{\times 4}$`,` $-1.924_{\times 2}$`,` $-1.696_{\times 2}$`,` $-1.635_{\times 4}$`,` $-1.282_{\times 2}$`,` $-1.241_{\times 2}$`,` $-1.021_{\times 2}$`, -0.780, -0.748,` $-0.040_{\times 2}$`,` $0.000_{\times 26}$`,` $0.987_{\times 4}$`, 2.132,` $3.814_{\times 2}$`,` $4.329_{\times 2}$`,` $5.762_{\times 2}$`, 7.146, 7.898, 7.899, 9.689`

#10: $\mathcal{P}/g^2 = -11.656854$, $U(1) \times U(1)$	
$m^2_{\rm scalar}L^2$	$-3.515_{\times 5}$`, -2.485,` $-2.121_{\times 4}$`,` $-1.409_{\times 4}$`, -1.286,` $-1.092_{\times 2}$`,` $-0.783_{\times 4}$`,` $-0.515_{\times 8}$`,` $0.000_{\times 26}$`, 2.485,` $3.268_{\times 4}$`, 4.801,` $5.652_{\times 4}$`,` $6.364_{\times 2}$`, 7.029,` $8.485_{\times 2}$

#11: $\mathcal{P}/g^2 = -12.000000$, $U(1) \times U(1)$ $\mathcal{N} = 1$	
$m^2_{\rm scalar}L^2$	$-2.250_{\times 4}$`,` $-2.229_{\times 4}$`, -2.196,` $-2.000_{\times 4}$`,` $-1.912_{\times 4}$`,` $-1.517_{\times 4}$`,` $-1.250_{\times 4}$`, -0.732,` $0.000_{\times 30}$`, 2.732,` $3.067_{\times 3}$`, 3.067,` $4.412_{\times 4}$`, 8.196,` $8.679_{\times 4}$

#12: $\mathcal{P}/g^2 = -13.623653$, $U(1)$	
$m^2_{\rm scalar}L^2$	$-2.812_{\times 2}$`,` $-2.714_{\times 2}$`, -2.595, -2.174, -2.056,` $-1.754_{\times 2}$`,` $-1.488_{\times 2}$`, -1.410, -1.374,` $-1.339_{\times 2}$`,` $-1.232_{\times 2}$`, -1.194, -1.159,` $-1.118_{\times 2}$`, -0.527,` $-0.480_{\times 2}$`,` $-0.300_{\times 2}$`,` $0.000_{\times 27}$`, 1.322, 1.789,` $3.995_{\times 2}$`, 4.024,` $4.668_{\times 2}$`, 5.203,` $5.736_{\times 2}$`, 5.861,` $7.007_{\times 2}$`, 7.073, 8.228, 11.974, 12.284`

#13: $\mathcal{P}/g^2 = -13.676114$, $\langle$No gauge symmetry$\rangle$	
$m^2_{\rm scalar}L^2$	`-3.007, -2.998,` $-2.621_{\times 2}$`, -2.301, -2.178, -2.068, -1.864, -1.862,` $-1.543_{\times 2}$`, -1.437, -1.389,` $-1.319_{\times 2}$`,` $-1.252_{\times 2}$`, -1.216, -1.101, -1.071, -0.973, -0.903, -0.102,` $0.000_{\times 28}$`, 0.391,` $1.073_{\times 2}$`, 1.257, 2.339, 3.688,` $4.914_{\times 2}$`, 5.471,` $5.549_{\times 2}$`, 5.984, 6.109, 6.991, 8.135, 8.531,` $10.188_{\times 2}$`, 11.360`

#14: $\mathcal{P}/g^2 = -14.970385$, $U(1)$

$m^2_{\text{scalar}}L^2$	-2.857, $-2.754_{\times2}$, $-2.430_{\times2}$, -2.130, -1.971, $-1.842_{\times2}$, -1.795, $-1.451_{\times2}$, -1.006, $-0.939_{\times2}$, $-0.884_{\times2}$, -0.849, $-0.741_{\times2}$, $0.000_{\times27}$, $0.257_{\times2}$, 1.063, 1.884, $2.892_{\times2}$, $3.311_{\times2}$, $3.937_{\times2}$, 4.303, 5.533, 7.717, $7.746_{\times2}$, $8.018_{\times2}$, 8.580, 8.683, 17.599, 17.612, 20.534, 20.564

#15: $\mathcal{P}/g^2 = -16.414456$, $\langle$No gauge symmetry$\rangle$	
$m^2_{\text{scalar}}L^2$	-2.975, -2.730, -2.667, -1.807, -1.790, -1.772, -1.593, -1.568, -1.550, -1.548, -1.345, -1.267, -1.245, -1.119, -0.907, -0.846, -0.793, -0.430, $0.000_{\times28}$, 0.338, 0.653, 1.439, 1.991, 2.335, 2.446, 2.470, 2.916, 4.174, 4.454, 5.969, 6.310, 7.583, 7.920, 7.945, 7.985, 8.175, 8.905, 9.267, 9.762, 10.891, 11.208, 14.959, 15.076

#16: $\mathcal{P}/g^2 = -17.876443$, $\langle$No gauge symmetry$\rangle$	
$m^2_{\text{scalar}}L^2$	-3.470, -3.196, -3.021, -2.739, -2.335, -1.826, -1.695, -1.641, -1.403, -1.252, -1.088, -1.076, -0.714, -0.665, -0.475, $0.000_{\times28}$, 0.043, 0.555, 1.418, 2.490, 2.508, 3.069, 3.231, 3.549, 3.865, 4.493, 4.913, 4.951, 5.148, 5.710, 6.431, 7.783, 7.855, 7.929, 7.972, 8.082, 8.863, 9.845, 11.638, 11.712, 12.309, 18.666, 18.751

#17: $\mathcal{P}/g^2 = -18.052693$, $\langle$No gauge symmetry$\rangle$	
$m^2_{\text{scalar}}L^2$	-3.358, -3.291, -2.830, -2.776, -2.307, -2.274, $-1.994_{\times2}$, -1.898, $-1.150_{\times2}$, -0.945, -0.796, -0.724, -0.686, -0.626, $-0.335_{\times2}$, $0.000_{\times28}$, 1.647, $2.012_{\times2}$, 2.290, 4.228, $4.270_{\times2}$, 5.563, $6.342_{\times2}$, 6.656, 6.896, 7.209, 7.323, $7.358_{\times2}$, 7.652, $8.033_{\times2}$, 8.269, 10.677, 10.966, 15.802, 15.892

#18: $\mathcal{P}/g^2 = -21.265976$, $\langle$No gauge symmetry$\rangle$	
$m^2_{\text{scalar}}L^2$	-4.227, -3.671, -3.029, $-2.320_{\times2}$, -1.955, -1.804, -1.267, -1.252, $-0.746_{\times2}$, -0.391, -0.364, $0.000_{\times28}$, 1.286, 3.114, 3.442, 3.871, 4.092, 4.180, $4.748_{\times2}$, $4.820_{\times2}$, 6.519, 7.816, 7.922, 8.240, $9.034_{\times2}$, 9.127, $10.227_{\times2}$, 11.045, 11.179, 11.535, $11.968_{\times2}$, 14.471, 18.868, $20.926_{\times2}$, 22.287

#19: $\mathcal{P}/g^2 = -21.408498$, $\langle$No gauge symmetry$\rangle$

$m^2_{\text{scalar}}L^2$	-4.447, -3.918, -3.905, -2.896, -2.408, -1.007$_{\times 2}$, -0.591, -0.468, -0.246, -0.093$_{\times 2}$, -0.004, 0.000$_{\times 28}$, 0.657$_{\times 2}$, 2.255, 3.408, 3.743, 4.224, 4.282, 4.765, 4.782, 5.812$_{\times 2}$, 6.202$_{\times 2}$, 6.438$_{\times 2}$, 7.719, 9.859, 10.305, 10.614, 12.377$_{\times 2}$, 12.537$_{\times 2}$, 14.437, 14.464, 16.669, 16.748, 28.669, 28.685
#20: $\mathcal{P}/g^2 = -25.149369$, $\langle$No gauge symmetry$\rangle$	
$m^2_{\text{scalar}}L^2$	-3.089, -2.400, -2.166, -1.825, -1.744, -1.481, -1.316, -1.268, -0.941, -0.670, -0.489, 0.000$_{\times 28}$, 0.750, 1.379, 1.605, 1.759, 1.920, 3.345, 3.382, 3.421, 3.822, 4.931, 5.930, 6.725, 7.256, 7.969, 7.998, 8.049, 8.210, 9.836, 10.366, 10.824, 11.837, 12.629, 12.725, 13.048, 13.863, 18.891, 19.022, 24.203, 24.239, 25.656, 25.669

Table B.1: Scalar masses of the stationary points #0 - #20 listed in [24], using the conventions of [13]

B.2 The $SO(4)'$ point

The following table contains the numerical data giving the location (in the conventions of [24, 25]), continous symmetries, and mass spectra of spin-3/2 and spin-1/2 fields that were used to set up an analytic calculation of the new $SO(4)'$-invariant critical point in Section 6.

Extremum: $V/g^2 = -8.807339$, Quality: $\lvert Q\rvert = 10^{-11.84}$, $\lvert\nabla\rvert = 10^{-4.45}$	
ϕ	$-0.3299_{[1247]+}$, $+0.3299_{[1257]+}$, $-0.3299_{[1347]+}$, $-0.3299_{[1357]+}$, $-0.4666_{[1678]+}$, $-0.4666_{[2345]+}$, $-0.3299_{[2468]+}$, $+0.3299_{[2568]+}$, $-0.3299_{[3468]+}$, $-0.3299_{[3568]+}$, $+0.2866_{[1236]-}$, $+0.2026_{[1247]-}$, $+0.2026_{[1257]-}$, $-0.2026_{[1347]-}$, $+0.2026_{[1357]-}$, $+0.2866_{[1456]-}$, $-0.2866_{[2378]-}$, $-0.2026_{[2468]-}$, $-0.2026_{[2568]-}$, $+0.2026_{[3468]-}$, $-0.2026_{[3568]-}$, $-0.2866_{[4578]-}$
Symmetry	[6-dimensional]
$(m^2/m_0^2)[\psi]$	2.049$_{(\times 8)}$
$(m^2/m_0^2)[\chi]$	4.098$_{(\times 8)}$, 3.774$_{(\times 8)}$, 0.391$_{(\times 32)}$, 0.007$_{(\times 8)}$

B.3 The locations of critical point #8 and #9

To 150 accurate digits, the 70-vector $\phi_{\mathtt{n}}$ that gives the 56-bein for critical point #8 according to

$$\mathcal{V}^{\tilde{\mathcal{A}}}{}_{\tilde{\mathcal{B}}} = \exp\left(\sum_{\mathtt{n}} \phi_{\mathtt{n}} g^{(\mathtt{n})}\right)^{\tilde{\mathcal{A}}}{}_{\tilde{\mathcal{B}}} \tag{C.1}$$

(using the conventions of [24]) is listed below – only nonzero entries are given, and index counting starts at 1. Furthermore, the following relations between entries have been employed to further shorten the presentation, i.e. entry ϕ_{02} is not listed as this is related in a simple way to ϕ_{01}, etc.:

$$\begin{gathered}\phi_{01} = \tfrac{1}{2}\phi_{02}\,, \qquad \phi_{06} = 2\phi_{07}\,, \qquad \phi_{08} = \phi_{09} = -\phi_{16} = \phi_{21}\,, \qquad \phi_{10} = -\phi_{15}\,, \\ \phi_{31} = -\phi_{34}\,, \qquad \phi_{32} = -\phi_{33}\,, \qquad \phi_{36} = \tfrac{1}{2}\phi_{37} = \phi_{38} = \phi_{45} = \phi_{50}\,, \\ \phi_{40} = \phi_{42} = \phi_{67} = -\phi_{68} = \tfrac{1}{2}\phi_{41}\,, \qquad \phi_{44} = -\phi_{51}\,, \qquad \phi_{65} = -\phi_{66} = \phi_{69} = \phi_{70}\,.\end{gathered} \tag{C.2}$$

		Critical point #8
ϕ_{01}	=	+0.08149800991420636144284179340047368401301087858848 42800995230168482979337459751986503947341102536696 59441836820594742957249335857968784123454183582584
ϕ_{03}	=	-0.03314238242442091820006312903856728674314315582873 33607526567504541404243330729534141329646921199538 45271667847520130139998373725460397478551252545118
ϕ_{04}	=	-0.22928078467725455928580984487808194151230806883443 52817043595346048767161580963041290553976047472470 09427009336229746194495419166858363204010872255404
ϕ_{05}	=	-0.10445671578332632889643239570691855166585035939984 09510559494413203503301949310072312320346077670660 27840429065891749231499259158427087655108299991042
ϕ_{06}	=	+0.02036735311060190149294505346424483818060735003475 33795924606519641760557682342896665913283892131149 53746151204446247731496900850004187893794272273321
ϕ_{08}	=	-0.03559865621956356146272131345605921475734803407844 80808453768177720686695293865553919293919403128531 60885826226938208398313742092471961391351069371766
ϕ_{10}	=	+0.13881820608352000126429425461999416939108789579709 31005256129004995171127854992746826585835114404814 11798589154652179505873190649683374924456901646435
ϕ_{30}	=	+0.02677921177333006507310803029303476490770449118409 72584501818311530093660524824522672899077535202704

		97803921727780281853237039415725239476439754669138
ϕ_{31}	=	-0.06942962363246406699044756399786065315982260999419 98966289903099323484896576064624324183385175162121 52624041595162697366413873359035893406104209038188
ϕ_{32}	=	-0.06750387272461459056792498795164290446838069222598 55102223202096333072069236412208655595135957933692 29229827936280560414372305216716684747899854200512
ϕ_{35}	=	-0.16563845903825819905400315828875607122734971117249 70517081624510177063453676953771321265847885526948 03052004918105676586064786133797026288648172745514
ϕ_{36}	=	+0.10378459150725177347878495867068630126872774257821 35237695229651616198582397594753191694042112778283 96565840528022497448214465709422149654857239137791
ϕ_{40}	=	+0.12073223619836159819915224622226067361300930438839 96035236216960804405540107886213345202334369260048 27377925737213842511361330833997851495203509305436
ϕ_{43}	=	+0.12684630664797953637045948007446536096364288507262 81804893417909807481600218573392242444093258304550 19824947445386212857704199239094827036377909217641
ϕ_{44}	=	+0.03017961664646812668987531418299021421338719589454 93792965565568997142878587951334137071978438388328 46228107448721261587957827445688491766425382855899
ϕ_{56}	=	-0.18720553994091578975021010844044578939041727686172 69390824549047801767357394476060516588050135081207 12281162342828736033619854130471810569228674929438
ϕ_{65}	=	-0.11016546490484786642658912198932195287609915468539 56525300344607258210753372221149767989284706041286 20213656425808941886997690051805968358012094919480

To 130 digits, the corresponding 70-vector for critical point #9 is given in the following table, where we have made use of these relations to shorten the presentation:

$$\begin{aligned}
&\phi_{11} = -\phi_{12} = -\phi_{17} = \phi_{18}\,, \qquad \phi_{21} = \phi_{24} = \phi_{25} = \phi_{28} = \phi_{29}\,,\\
&\phi_{36} = \phi_{42}\,, \qquad \phi_{37} = \phi_{41}\,, \qquad \phi_{38} = \phi_{40}\,,\\
&\phi_{43} = -\phi_{46} = -\phi_{47} = -\phi_{52} = -\phi_{53} = -\phi_{56} = \phi_{59} =\\
&\quad = -\phi_{60} = -\phi_{63} = \phi_{64} = -\phi_{65} = \phi_{70}.
\end{aligned} \tag{C.3}$$

		Critical point #9
ϕ_{01}	=	-0.1483853086359978954346538436081596890045505125812994603695611582 96912659174620420013261089599470046207901483798616258605178379830
ϕ_{02}	=	-0.29677061727199579086930768721631937800910102516259892073912231659 3825318349240840026522179198940092415802967597232517210356759659
ϕ_{03}	=	-0.25732646426692985616707435046847694741517689249474313726724786427 2674663810312601338119308768781920952354455078216408698296166166
ϕ_{04}	=	-0.21788231126186392146484101372063451682125275982688735379537341195 1524009271384362649716438338623749488905942559200300186235572673
ϕ_{05}	=	-0.16737947827750216832085744976228405120097892770824596184306966143 4570173530882288314997735806904725069736607758864110903053338179
ϕ_{06}	=	-0.11687664529314041517687388580393358558070509558960456989076591091 7616337790380213980279033275185700650567272958527921619871103685
ϕ_{07}	=	-0.05843832264657020758843694290196679279035254779480228494538295545 8808168895190106990139516637592850325283636479263960809935551843
ϕ_{08}	=	-0.08359029866858233034547941217571396802759842476707237180476077865 8414463573049930973956617579727777325402250717989610457138556824
ϕ_{11}	=	+0.11635037784601938286230192233288778591954537037069254009954531733 0228235640944330444860413700610898200934371266006226890321421623
ϕ_{21}	=	-0.09047325343654872016078886417723869587535769000540920521539879642 5492592428866196558540192536451998332250749011873992524792510049
ϕ_{30}	=	+0.11529784295177731823315799539079618659722591993286848051710413015 5452031342072563374023174551461293301668567880962837431222057500
ϕ_{35}	=	-0.10946742307805299304699247033136305807178610513235570668890729956 3150106785128064860276838743886677194085872972121844822667468398
ϕ_{36}	=	+0.24667108042473646307996708755925237239714154731457784561932927971 9762167903809320049487455179596894551403218163281674880317791563
ϕ_{37}	=	+0.49334216084947292615993417511850474479428309462915569123865855943 9524335807618640098974910359193789102806436326563349760635583126
ϕ_{38}	=	+0.41111846737456077179994514593208728732856924552429640936554879953 2936946506348866749145758632661490919005363605469458133862985938
ϕ_{39}	=	+0.32889477389964861743995611674566982986285539641943712749243903962 6349557205079093399316606906129192735204290884375566507090388750
ϕ_{43}	=	-0.08222369347491215435998902918641745746571384910485928187310975990 6587389301269773349829151726532298183801072721093891626772597187

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